Intereference management in cognitive radio networks

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INTEREFERENCE MANAGEMNET IN COGNITIVE RADIO NETWORKS

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This thesis is dedicated to my parents for their love and endless support.

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Abstract

Radio spectrum is becoming increasingly scarce as more and more devices go wireless. Meanwhile, studies indicate that the assigned spectrum is not fully utilised. Cognitive radio technology is envisioned to be a promising solution to address the imbalance between spectrum scarcity and underutilisation. Cognitive radio enables the unlicensed (secondary) user to establish a communication link in licensed (primary) spectrum on the condition that there is no or minimal interference to the primary user.

The interference management has become an important topic in cognitive radio in order to manage and fulfill the regulatory constraints. The management of interference is, unquestionably, required to treat and quantify all the interference produced by the cognitive transmission at the primary users. In order to manage this interference, the secondary users must be able to adjust their parameters to fulfill these constraints.

In addition, the performance of contemporary multicell wireless networks is limited by intercell interference (ICI), due to cochannel transmission in other cells. This performance degradation is especially severe for users close to the cell-edge. As a solution, in this thesis different cognitive beamfomring techniques are proposed by deploying cognitive cells on the primacy cells boundaries to support the primary cell-edge users as well as servicing the secondary users.

This thesis proposes interference management techniques based on cognitive beamforming in a cellular network. We have identified conditions and proposed different techniques for optimal usage of radio spectrum, by allowing coexistence on the same spectrum resources between primary and cognitive users. Deploying cognitive cells on the primary cells borders results to ICI mitigation for primary cell-edge users within the cognitive cell and also leads to supporting cognitive users with the same allocated spectrum to the primary network. The aim is to minimise the total transmit power across the cognitive network while maintaining the required signal-to-interference-plus-noise ratio (SINR) for all primary cell-edge/cognitive users within the cognitive cell and to mitigate the interference caused by the cognitive system towards the primary users. It forms the fundamental basis for interference management in cognitive radio systems and consequently gives insights into the design and deployment of cognitive radio networks. At the end, we introduce robust cognitive beamforming based on imperfect channel state information for both primary and cognitive users' channel. These different approaches to interference management at cognitive radio networks contribute to the increasing set of techniques that will make cognitive radio possible to deploy.

Abbreviations

AoD	angle of departure
BER	bit error rate
BS	base station
CBF	coordinated beamforming
CC	cognitive cycle
CDF	cumulative distribution function
CDMA	code division multiple access
CR	cognitive radio
CRS	cognitive radios
CSI	channel state information
CSIT	channel state information at transmitter
DAS	distributed antenna system
DBF	decentralised beamforming
DCP	disciplined convex programming
DDA	distributed-array antenna
DF	decode-and-forward
DPC	dirty paper coding
DS-CDMA	direct-sequence code division multiple access
EDGE	enhanced data rates for GSM evolution
ETSI	European telecommunications standards institute
FCC	federal communications commission
FDD	frequency division duplexing
GSM	global system for mobile
ICI	intercell interference
IID	independent and identically distributed
IMT	international mobile telecommunications
INR	interference plus noise ratio
ISO	international standards organization
ITU	international telecommunication union
LMI	linear matrix inequality
LoS	line of sight
LP	linear programming
LTE	long term evolution
MBF	multicell beamforming
MIMO	multiple input multiple output

LIST OF ABBREVIATIONS

MMSE	minimum mean squared error
M-QAM	multi-level quadrature amplitude modulation
MS	mobile station
OFDMA	orthogonal frequency-division multiple
OFDMA	orthogonal frequency-division multiple access
OSA	opportunistic spectrum access
PDA	personal digital assistant
PDF	probability density function
PU	primary user
QCLP	quadratically constrained linear program
QCQP	quadratically constrained quadratic program
QoS	quality of service
QP	quadratic program
QPSK	quadrature phase shift keying
RF	radio frequency
RS	relay station
SDP	semidefinite programming
SDR	software defined radio
SINR	signal to interference plus noise ratio
SIR	signal to interference ratio
SNR	signal-to-noise ratio
SOC	second order cone
SOCP	second order cone programming
SU	secondary user
TDD	time division duplexing
UHF	ultra high frequency
UPA	user position aware
ZF	zero forcing
ZMCSCG	zero mean circularly symmetric complex Gaussian

Symbols

a	scaler a
$\mathbf{a}$	vector $\mathbf{a}$
$\mathbf{A}$	matrix $\mathbf{A}$
$\mathbf{w}$	beamforming weight vector
$\lvert a \rvert$	magnitude of a
$\mathrm{Re}\{a\}$	real part of a,
$\lVert \mathbf{a} \rVert_l$	l^{th} norm of $\mathbf{a}$
$\lVert \mathbf{a} \rVert_2$	Euclidean norm of $\mathbf{a}$
$\lVert \mathbf{A} \rVert$	Frobenius norm of $\mathbf{A}$
$\mathbf{A}^*$	complex conjugate of $\mathbf{A}$
$\mathbf{A}^T$	transpose of $\mathbf{A}$
$\mathbf{A}^H$	complex conjugate transpose of $\mathbf{A}$
$[\mathbf{A}]_{i,j}$	$(i,j)^{th}$ entry of $\mathbf{A}$
$\mathrm{Tr}\,(\mathbf{A})$	trace of $\mathbf{A}$
$\mathrm{vec}\,(\mathbf{A})$	stacks $\mathbf{A}$ into a vector columnwise[1]
$\mathbb{E}\,(\cdot)$	expectation operator
$\mathbf{A} \succeq 0$	$\mathbf{A}$ is a positive semidefinite matrix
$\mathbf{A} \succeq \mathbf{B}$	$\mathbf{A} - \mathbf{B}$ is a positive semidefinite matrix
$\mathbf{a} \succ 0$	all elements of $\mathbf{a}$ are positive
$\mathbf{a} \succeq 0$	all elements of $\mathbf{a}$ are nonnegative
$\mathbf{a} \succ \mathbf{b}$	element-wise greater than
$\mathbf{a} \succeq \mathbf{b}$	element-wise greater than or equal to
$\mathbf{I}$	identity matrix with a suitable size
$\mathbf{e}_i$	column unit vector with a suitable size which contains all zeros except a one at the ith element
$\approx$	approximately equal to
$\mathbb{C}^n$	n-dimensional real vectors
$\mathbb{H}^n$	n-dimensional complex vectors
$\mathbb{R}^n$	n-dimensional complex Hermitian matrices
$\mathcal{CN}(\mu, \sigma^2)$	complex scalar Gaussian random distribution with mean μ and variance σ^2
C	convex set (convex cone)
$\mathbf{S}^n{}_+$	set of positive semidefinite matrices

[1]If $\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n \end{bmatrix}$ is $m \times n$, then $\mathrm{vec}\,(\mathbf{A}) = \begin{bmatrix} \mathbf{a}_1^T & \mathbf{a}_2^T & \cdots & \mathbf{a}_n^T \end{bmatrix}^T$ is $mn \times 1$.

LIST OF SYMBOLS

$\Pr\{A\}$	probability of an event A
ICDF(.)	inverse cumulative distribution function of the Chi-square random variable with m degree of freedom
$x \to a$	x approaches to a
rank(.)	rank of a matrix
$s.t.$	subject to
Σ	summation

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Chapter 1

Introduction

Reliable and fast wireless data transmission is emerging as a global phenomenon and becoming a major consideration in our lives such as internet, online shopping, and social networking. This has caused an exponential increase in the demand for the radio frequency spectrum. However, due to the huge growth of all wireless technologies, the radio spectrum is beginning to be crowded. Nevertheless, it has been found that the major licensed bands, such as those allocated for television broadcasting, amateur radio, paging, etc. are underutilised and some of the remaining bands are heavily used. This fact leads to a wastage of spectrum. Therefore, new techniques are needed to take advantage of the spectrum opportunities causing a reasonable level of interference in the licensed system. In addition to spectrum sensing algorithms, sharing protocols, policies, among other things, interference management has also become an important topic in cognitive radio in order to manage and fulfill the regulatory constraints.

1.1 Towards cognitive radio

1.1.1 Software defined radio

The availability of high computational capacity but low cost flexible hardware technologies such as Programmable Logic Devices, Digital Signal Processors, and Central Processing Units has opened new horizons in which non-military communication systems can be designed and managed. It was only in 1991 that Joseph Mitola coined the term software defined radio (SDR) and anticipated the feasibility of theoretical software design of communication systems within a short time span. Currently available radio devices are equipped with various standards such as GSM, EDGE, Wi-Fi, Bluetooth, LTE, etc. Specific dedicated electronic chain for switching from one standard to another which is known as the Velcro approach is required. The design and development of these radio devices has become a real challenge due to high degree of flexibility which urgently is required. Recent advances in hardware offer design possibility by which at least partially, software solutions can replace hardware signal processing devices that were required in the past. This is vividly a step closer to the anticipated SDR systems. Moreover, other debatable SDR system definitions exist. The software related radio concepts of SDR system as agreed by SDR Forum [9] defines software defined radio as *radio in which some or the entire physical layer functions are software defined.* The physical layer and software defined terms are respectively described as:

- Physical layer: *This is the layer within the wireless protocol in which processing of Radio Frequency, Intermediate Frequency, or baseband signals including channel coding occurs. According to ISO 7-layer model it is the lowest layer adapted for wireless transmission and reception.*

- Software defined: *This refers to the use of software processing within the radio system or device with operating implementation function and not controlling function.*

Thus, SDR systems are solely defined from the design and the implementation point of views. Consequently SDR appears to be a simple evolution form of the usual hardware radio systems. However, with the added software layer, enables the current technologies to control a large set of parameters if adaptation of radio equipment to their communication environment such as bandwidth, modulation, protocol, and power level are required. In order to control and optimise the reconfigurable radio devices, optimisation criteria related to the equipment hardware capabilities for the user and regulator must be defined. Introduction of autonomous optimisation capabilities in radio terminals and networks forms the basis of cognitive radio, a term which has also been suggested and coined by Joseph Mitola [1], [10].

1.2 The rise of cognitive radio

Joseph Mitola defined cognitive radio (CR), in his PhD dissertation as follows [10]

The term cognitive radio identifies the point at which wireless Personal Digital Assistant (PDA) and the related networks are sufficiently computationally intelligent about radio resources and related computer to computer communication to

1. *Detect user communication needs as a function of use context, and*
2. *Provide radio resources and wireless services most appropriate to these needs.*

Thus, this new concept autonomously meets the users' expectations such as profit maximisation in terms of Quality of Service (QoS), throughput or power efficiency without compromising the efficiency of the network. This involves the distribution of the required intelligence efficiently operating in both the network and the radio device.

In order to fulfill these requirements, J. Mitola introduced the notion of cognitive cycle (CC) as described in Fig. 1.1 ([1], [10]). The cognitive cycle is empowered with the capacity to collect information from the surrounding environment and digest the information for the purpose of the best learning, decision making, and tools prediction while considering constraints and the available information. An environment can refer to

- Geolocation.
- Spectrum occupation.
- Interference level (or interference temperature).
- Noise level uncertainty.
- Regulatory rules.

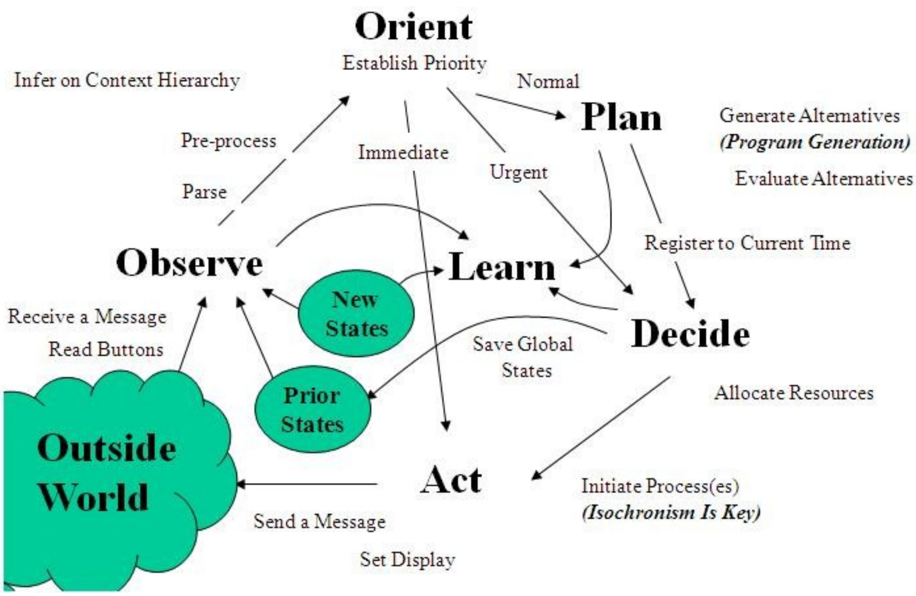

Figure 1.1: Cognition cycle introduced by Joseph Mitola [1].

In this work in depth discussion of reconfiguration of radio equipment is not provided, however it is generally accepted that SDR technology is needed to support cognitive radio [11].

1.3 Cognitive radio definition and characteristics

Since the original definition suggested by Mitola, several other definitions were proposed to define the edges of cognitive radio. The following few paragraphs provide the principal alternative definition of cognitive radio found in the literature.

In 2005, F. K. Jondral [12] suggested a definition that insists on one hand, on a tight relationship between SDR technologies and CR paradigm and on the other hand, on the importance of information exchange among different CRs. This definition however keeps its generality and do not seem to tackle a particular application.

1. Definition 1: Cognitive radio, by F. K. Jondral [12]

   A CR is an SDR that additionally senses its environment, tracks changes, and reacts upon its findings. A CR is an autonomous unit in a communication environment that frequently exchanges information with the networks it is able to *access as well as with other CRs.*

Whether CR is necessarily based on SDR devices is still a matter of debate. The evolution from SDR to CR is relevant to the fact that CR can be regarded as a paradigm to the design of a general purpose decision making engine. This evolution necessitates flexibility of the strategies of the running equipments. Advantages and disadvantages are associated with this approach. While CR provides a general optimiser that can transform any adaptable radio into a decent cognitive radio, it recurrently must be updated to face new radio designs and capabilities. Such tendency eventually could lead to very complex and heavy systems that would probably be underutilised by the host radio equipment. This indicates the importance of dimensioning of the decision making capabilities.

F. K. Jondrals definition further stresses the importance of communication and information exchange between a CR and surrounding environment, viz., the accessible networks and other CRs. Information exchange is the synonym of communication overhead and loss of throughput. Time and energy is spent correctly if the information exchange enables interference mitigation and avoids conflicts. This point is comprehensively dealt with in later chapters when multi-user CR networks are discussed. In this work general scenarios where collaboration among the CR users such as interference avoidance policies when accessing frequency bands resources are addressed. This does not necessarily imply information exchange.

During the same year 2005, the Federal Communications Commission (FCC) [13], in the United-States, and Simon Haykin [2], respectively, proposed more pragmatic definitions that described cognitive radio as the possible means that enables better use of spectrum.

2. Definition 2: Cognitive radio, FCC 2005 [13]

A cognitive radio is a radio that can change its transmitter parameters based on interaction with the environment in which it operates.

3. Definition 3: Cognitive radio, S. Haykin 2005 [2]

   Cognitive radio is an intelligent wireless communication system that is aware of its surrounding environment, i.e., its outside world, and uses the methodology of understanding-by-building to learn from the environment and adapt its internal states to statistical variations in the incoming Radio Frequency (RF) stimuli by making corresponding changes in certain operating parameters (e.g. transmit power, carrier-frequency, and modulation strategy) in real-time, with two primary objectives in mind: highly reliable communications whenever and wherever needed and efficient utilisation of the radio spectrum. This definition in general refers to parameters such as transmission frequency, modulation scheme, and either bandwidth or power allocated to each user as shown in Fig. 1.2.

   More recently, in 2009, the International Telecommunication Union (ITU) [14] also suggested a general definition, that appears to synthesize both the definitions proposed by the normalisation task force P1900.1 and the European Telecommunications Standards Institute (ETSI) [11].

4. Definition 3: Cognitive radio, ITU

   Cognitive radio System (CRS) is a radio system employing technology that allows the system to obtain knowledge of its operational and geographical environment, established policies, and its internal state; to dynamically and autonomously adjust its operational parameters and protocols according to its obtained knowledge in order to achieve predefined objectives; and to learn from the result obtained.

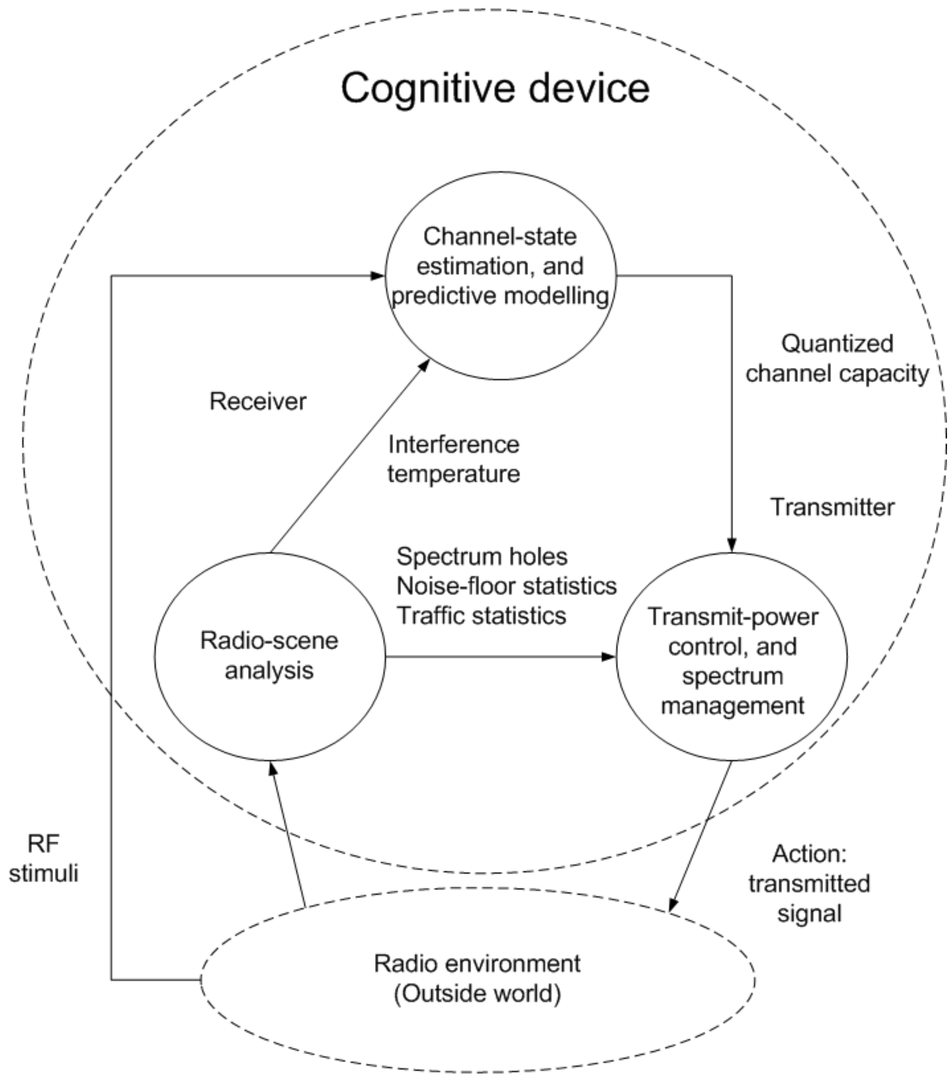

Figure 1.2: Cognition cycle introduced by Simon Haykin, where it shows the different states that any cognitive radio device must be able to follow by sensing the radio environment [2].

1.4 Motivation behind cognitive radio

The phenomenal surge of interest in this relatively new field is due to the fact that cognitive radio not only meets a pressing end by utilisation of precious natural resources, i.e., the radio spectrum, but also is a multi-challenging alternative to the other available wireless technologies. Fig. 1.3 shows the comparison between wireless networks and cognitive radio networks.

Cognitive radio provides novel solution for spectrum underutilisation. The solution is based on sensing of the radio environment with dual objectives as followings

1. Identification of the underutilised sub-bands of the radio spectrum that are not employed by the primary (legacy) users.

2. Providing the means for the underutilised sub-bands to be employed by unserviced secondary users.

The system autonomously achieves the above objectives. Multi-user cognitive radio networks ought to be self-organised. Additionally in order to limit the interference produced by secondary user, there would have to be a paradigm shifts from transmitter-centric wireless communications to receiver-centric as a new mode of operation. The underutilised frequency bands of the radio spectrum, owned by legally licensed (primary) users, are referred to as spectrum holes.

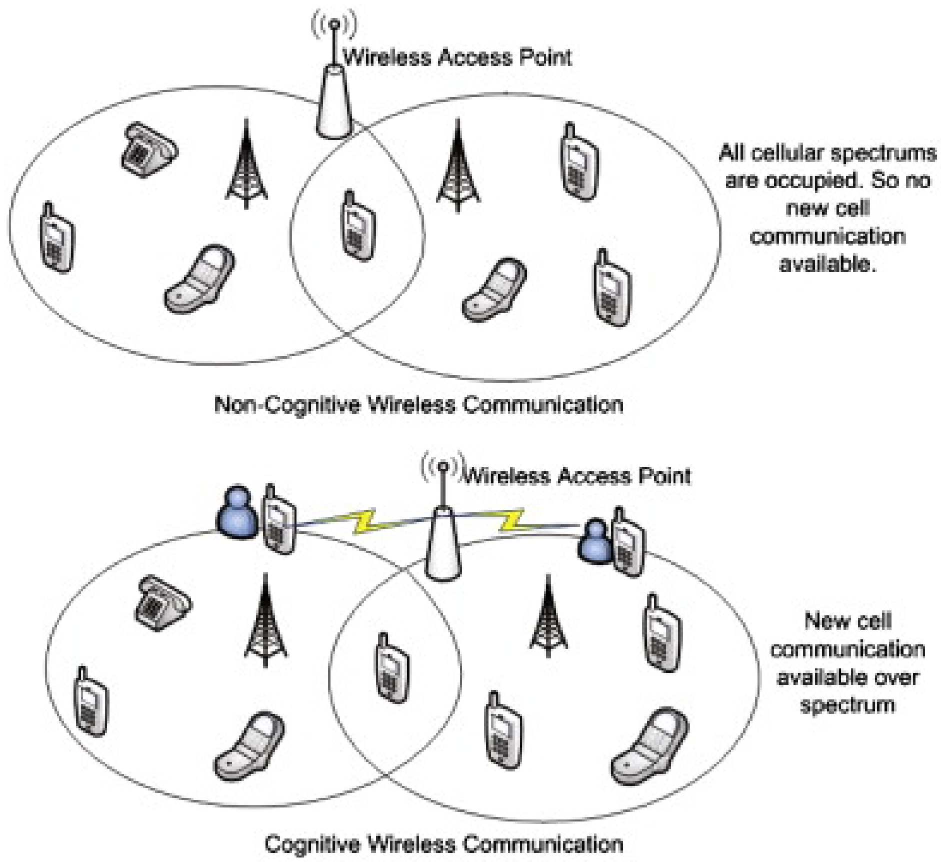

Figure 1.3: Difference between wireless networks and cognitive radio networks.

1.4.1 Spectrum holes

A spectrum hole is defined [2] as a band of frequencies assigned to a primary user, with condition that the assigned band is not utilised by the user at a particular time and specific geographic location. The operation of cognitive radio is pivotal on the availability of spectrum holes. Identification and exploitation of spectrum holes is technically confronted both by computer software and signal processing and communication technology. The stochastic occurrence of spectrum holes further complicates these technical challenges. Through real-time interaction with the radio environment, the spectrum holes at a specific time or location can be identified as shown in Fig. 1.4.

There are five major functional blocks of cognitive radio as

1. Spectrum sensing: This provides estimation of average power content and detection of spectrum holes.

2. Predictive modeling: This provides prediction of the time span that spectrum hole is likely to remain available for employment by secondary user.

3. Transmit power control: Where the data rate of each user is maximised subject to power constraint. This research is based on transmit power control, which will be continued in later chapters.

4. Dynamic spectrum management: Subject to usage cost, this ensures that the spectrum holes are fully distributed among secondary users.

5. Packet routing: This is a self organised scheme which is designed to route the packets across the radio network.

The primary objective of the research is to provide highly reliable communication for all users of the network while facilitating in a fair manner an efficient utilisation of the radio spectrum. The emergent behaviour of cognitive radio networks are seemingly irreducible phenomena where this phenomena not explicitly programmed. The

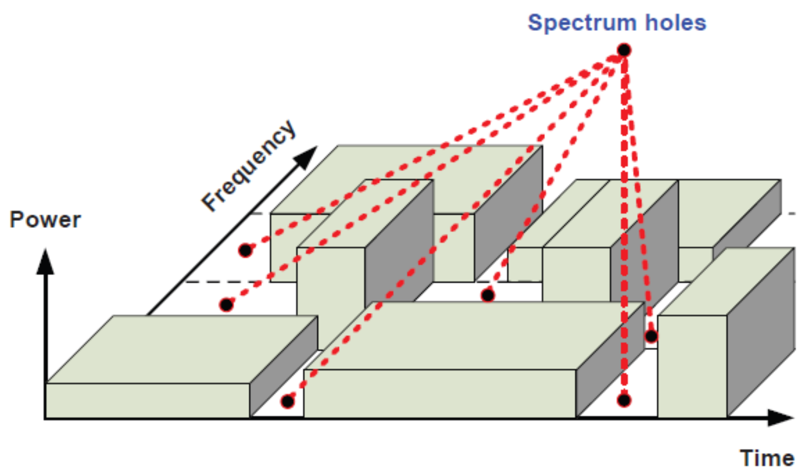

Figure 1.4: Spectrum hole concept.

emergent behavior of cognitive radio networks can be defined as positive or negative. The positive emergent behaviour is described as harmonious and efficient utilisation of the radio spectrum by both primary and secondary users of the cognitive radio, i.e., cooperation with or without minimal coordination. Characteristics such as disorder, traffic jams, chaos, and unused radio spectrum are associated with negative emergent behaviour.

The interference temperature is advantageous in quantification and management of interference source at the receiver's input. The interference temperature threshold depicts the worst characteristic of the radio environment. CR enables the secondary user to establish transmission links in vacant primary user channels in such manner that primary users are subjected to no or minimum interference. CR functionalities such as spectrum sensing, spectrum access, spectrum allocation, and spectrum management among various secondary users, and a reconfigurable hardware ensures realisation of the above operation [10], [2]. Interference temperature which is the widely used technique in detection of spectrum hole was introduced by FCC in 2003. This technique quantifies and measures interference in a given spectrum band in a particular location. The magnitude of this function is used as criteria to detect spectral holes.

In order to determine the spectral holes, spatial variation of the interference temperature is carried out by scanning of the radio environment using large number of sensor nodes. The radio environment is sensed by measuring the level of incoming RF stimuli. Thus, the sensed RF spectrum based on interference level falls into following spaces:

- Black spaces: These bands are occupied by high powered interferes.
- Gray spaces: These bands are partially occupied by low-powered interferes.
- White spaces: These bands are free from any RF interference, except for thermal noise and noise from lightening, etc.

Gray and White spaces are more desirable due to their low and acceptable interference temperature. Since the RF stimuli are non-stationary in space and time. Therefore, spatial and temporal characteristic of the signal are both involved in the estimation of interference temperature. Thus in relation to spatial-temporal variation of RF stimuli, quantification of spectral hole detection can be carried out by detection statistics. This is a parameter which denotes the dwell of a RF spectrum a white space. Alternative methods of radio scene analysis are featured by detection, matched filter, and energy detection. The advantage of FCC recommended interference temperature based spectrum sensing over the alternative methods lies in the fact that this method strictly does not exceed interference temperature threshold set by the primary user. Various secondary user transmission modes of secondary user have either been discussed or proposed in the literature. These can be broadly categorised into interference avoidance, i.e., white space, and the interference management, i.e, black and gray space modes, [2]. In [15], the aggregate effect was taken into account and complex stochastic models were built to characterise the exact PDF (probability density function) of the accumulated interference power. Moreover, the interference avoidance ability of cognitive radio transmitters was considered by introducing the concept of an exclusion region. As illustrated in Fig. 1.5 an exclusion region is defined as a disk centered at a primary

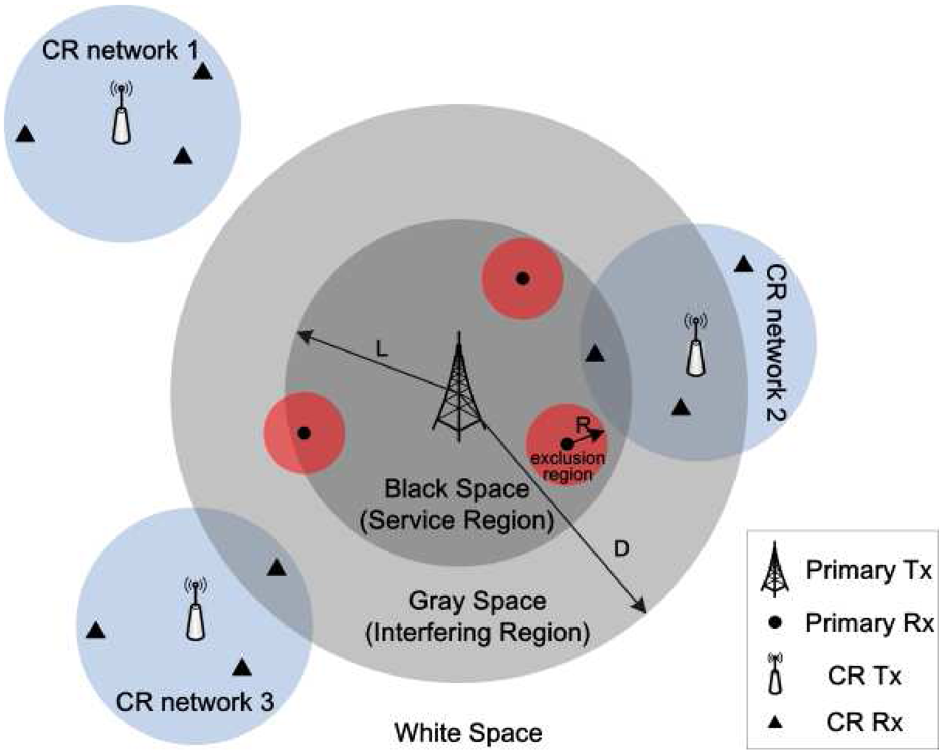

Figure 1.5: Coexistence of a primary network and randomly distributed CR networks with illustrations of the exclusion region, black space (service region), Gray space (interfering region), and white space [3].

receiver with a radius R. Any cognitive radio transmitter within the exclusion region is regarded as a harmful interferer and is therefore forbidden to transmit.

When the locations of cognitive radio transmitters follow a Poisson point process [16] with a density λ, the PDF of the aggregate interference can be computed as a function of R. As shown in Fig. 1.6, it is found that a slight increase of R can effectively reduce both the mean and variance of the received interference power. The cognitive-primary interference modelling is further extended in [15] by taking into account interference management schemes for cognitive radio networks including power and contention control.

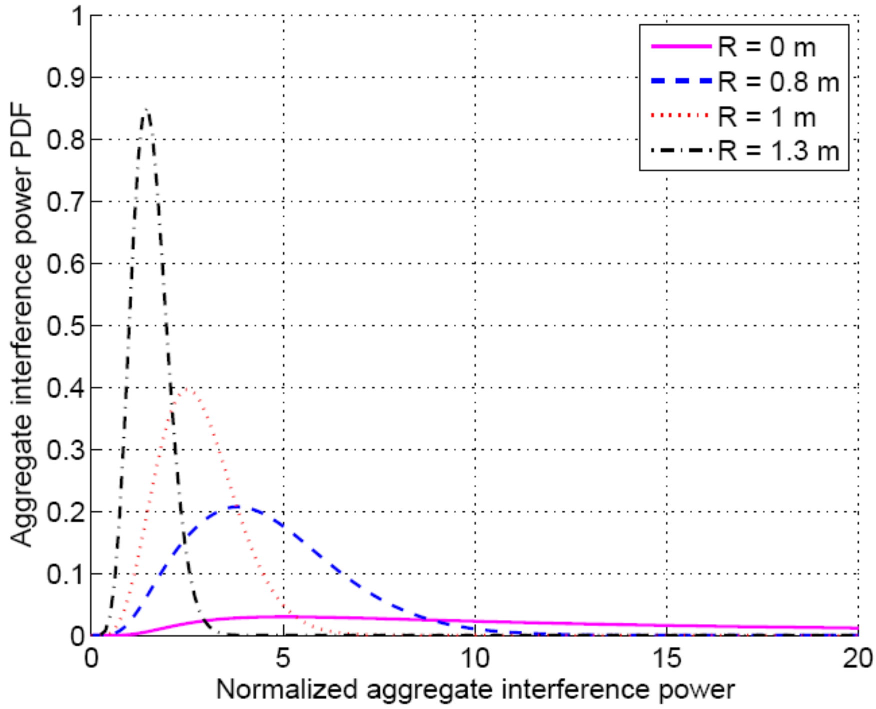

Figure 1.6: PDFs of the aggregate interference power (normalised to the transmit power of the interferers) with different values of the exclusion region radius R (CR transmitter density λ=1)[3].

1.4.2 Interference avoidance mode

In this mode often termed as interweave access, sensing of the radio frequency spectrum by secondary user results in finding of the spectrum holes which corresponds to white space. The presence of spectrum holes in the primary user channels are highlighted in Fig. 1.7. For transmission purposes these spectrum holes are used by the secondary user. This scheme is often referred to as opportunistic spectrum access (OSA), where no concurrent transmission of the primary and secondary users takes place. Upon reappears of the primary user, the secondary user vacates the channel. The secondary user connection is terminated if there is no other available channel. Since the secondary user exerts no control over the resource availability, the transmission of the secondary user is blocked when the channel is occupied by the primary user. The forced termination and blocking of a secondary user connection is shown in Fig. 1.8. The forced termination and blocking probabilities are the principal parameters which determine both the throughput and viable existence of the secondary user. The forced termination is dependent up on traffic behaviour of the primary and the secondary users including arrival rates, service time, etc. In the case of multiple secondary user groups with different traffic statistics, the forced termination and blocking probabilities result in unfairness amongst the secondary user groups which leads to a difficult QoS provisioning task.

1.4.3 Interference management

The interference management mode in a CR can be described as either overlay or underlay, both of which allow concurrent transmission.

1.4.4 Overlay mode

The Overlay mode refers to a technique in which the secondary user exploits additional knowledge of the primary user transmissions. Transmission opportunities are increased by enhancement of the primary user messages, and reliance on dirty paper coding techniques to mitigate interference at the receiver [17]. This access mode has also been referred to as CR enabled cooperative relaying [18].

1.4.5 Underlay mode

The Underlay mode refers to a technique in which the secondary user can only share the spectrum provided its signal remains below the acceptable interference threshold of the primary user. The threshold refers to the tolerated peak or average power of the primary user receiver [19], [20]. This is a useful constraint when the signal variation at the primary user receiver is quasi-static, such as Television unit. However, when signal variations from the primary user transmitter to the primary user receiver are random, outage probability is a better measure. Outage at the primary user receiver occurs when the SINR falls below a specific threshold which is different from the interference threshold. The spectrum sharing constraint in this scenario is based on acceptable

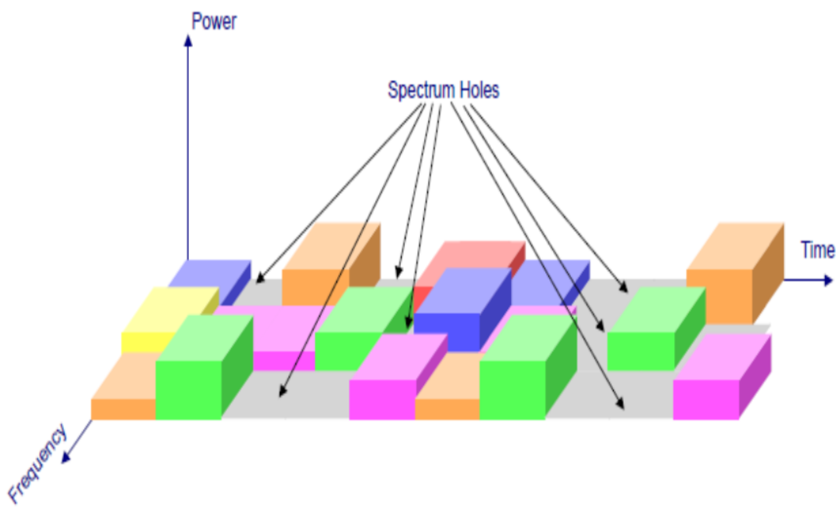

Figure 1.7: Conceptual view of spectrum holes.

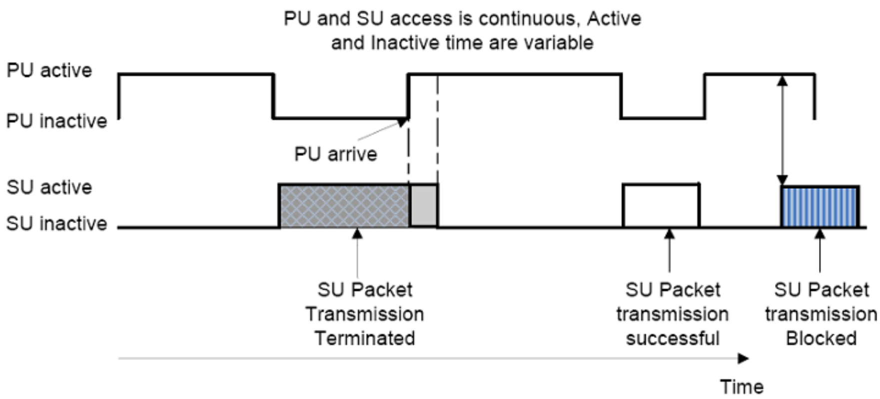

Figure 1.8: Illustration of forced termination and blocking.

long or short term outage to the primary user network. The secondary user transmit power is the key factor for coexistence of the secondary user in an underlay mode. The coverage area of the secondary network in an underlay mode due to strict transmit power constraint, is severely restricted [17]. Calculation of secondary user transmit power generally requires knowledge of primary user receiver location and the channel gains between the secondary user transmitter and primary user receiver and vice versa. The secondary user operates independent of the primary user network. Thus estimation of the parameters is required as these are often unknown to the secondary user transmitter. The estimations are achieved at the expense of higher signaling overhead.

In underlay mode it is often necessary to derive the signal-to-interference-plus-noise ratio (SINR) distribution, in order to calculate practical performance measures such as capacity and SINR outage [21], [22]. The propagation channel between the primary user and the secondary user is generally subject to fading which is traditionally modelled by a gamma or log-normal distribution [23]. The cumulative distribution function (CDF) for the signal-to-interference ratio (SIR) is often known in closed form [20]. However, in the case of SINR, which include the effect of noise, these models yield integrals for which closed form expressions are not known. Although integral inequalities [24] have successfully been applied to obtain bounds on capacity problems [25], [26], their application to SINR have been almost non-existent.

Fig 1.9 depicts the underlay and overlay dynamic spectrum access where these paradigms fit into what is commonly known as hierarchical-access schemes, referring to the fact that secondary users need to fulfill the constraints imposed by the primary user.

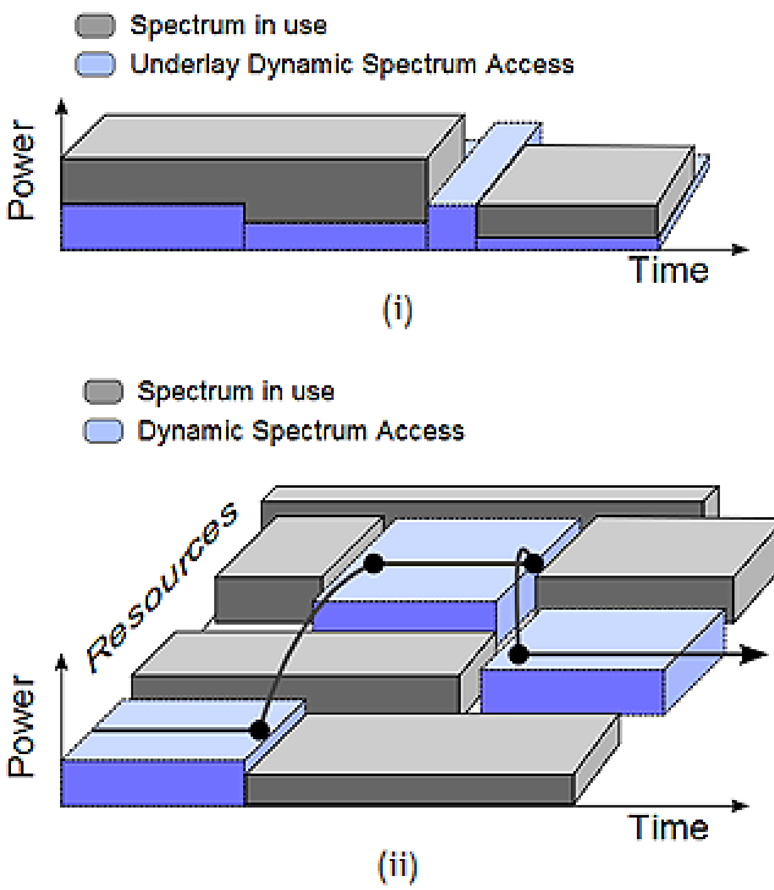

Figure 1.9: Hierarchical-access paradigms: (i) underlay dynamic spectrum access, (ii) overlay dynamic spectrum access.

1.5 Radio resource management in a cellular concept

In order to offer sophisticated mobile communications over a large area, wireless cellular networks divide the covered area into cells, as shown in Fig. 1.10. All communications within each cell are served by one base Station (BS) located in the cell-center. The same frequency resource is repeatedly available (reused) for other cells. Hence, the main advantage of using cellular systems is that through reusing radio channels in cells, the network coverage can be provided to areas of any size.

However, how to determine the size and the shape of a cell, as well as how to allocate resources among cells are very important in radio network planning, as they

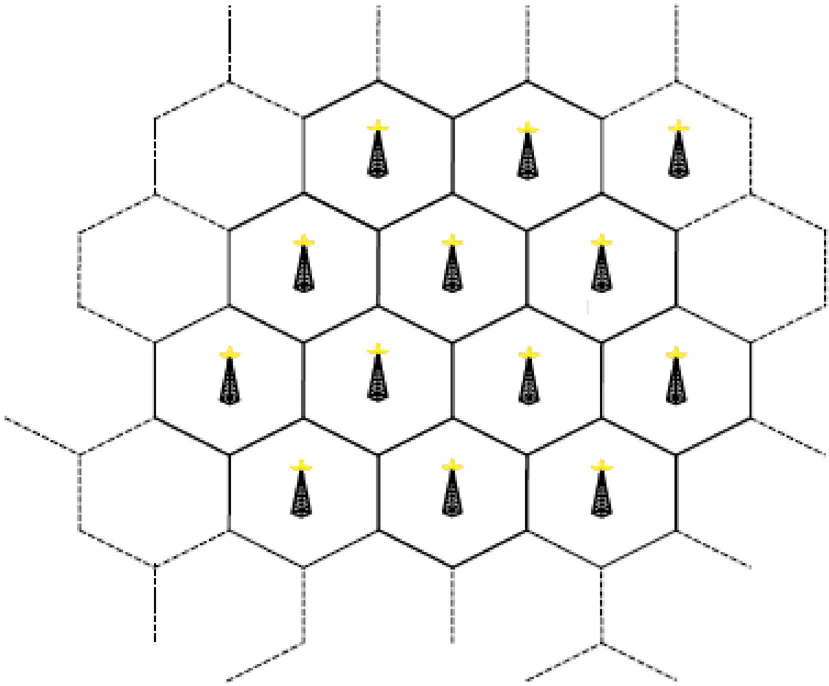

Figure 1.10: Illustration of a cellular system, where each cell is served by one BS in the cell-center.

may largely influence the system performance. The size and the shape of each cell depend on signal quality received within the covered cell-area, which is related to many factors, such as the surrounding terrain, buildings, the height of transmission antennas, the transmission power of the BS, the expected traffic demands and density, as well as the atmospheric conditions, etc. Cells are generally represented as idealised regular hexagons, but because of topographical and environmental conditions, this is only an approximation of what actually occurs [27]. Naturally, in a real world scenario, the cell shapes are very irregular and overlap with each other by approximately 10 to 15%. This enables users operating near the boundary of a cell to choose which BS they are associated to.

Enhancing the cell coverage by allowing as many users to communicate reliably irrespective of their location and mobility appears to be a primary concern of network service providers. This task is typically fulfilled by doing aggressive spectrum reuse which on one side enhances the spectral efficiency, whereas on the other side it causes

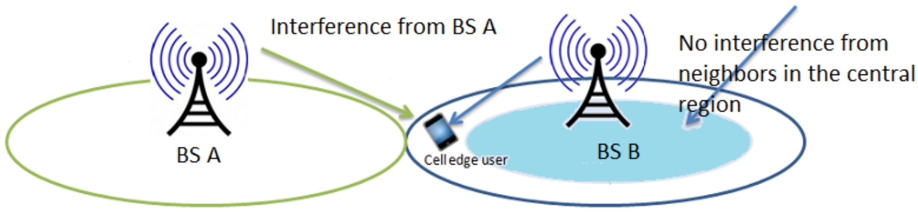

Figure 1.11: A typical two-cell layout, where the cell-edge user is interfered by BS A from the neighboring cell [4].

severe intercell interference (ICI) among the users of same spectrum, particularly cell-edge users located close to the cells boundaries as shown in Fig. 1.11.

Radio resource management has been evolved as an efficient tool to coordinate, mitigate and manage ICI while enhancing the network performance in a cellular networks. The incurred ICI in cellular networks with universal frequency reuse is severe and highly random due to its dependence on the channel statistics and on the dynamics of the multiuser scheduling decisions. Therefore, it is important for the system designers to accurately characterise the behavior of the ICI in order to quantify various network performance metrics and to develop efficient resource allocation and interference mitigation schemes. More specifically, efficient spectrum/subcarrier allocation and power control management solutions are needed to leverage the potential of cellular networks.

1.6 Transmit power control and dynamic spectral management

The feedbacks obtained from radio scene analysis are used at the CR transmitter for control of power and management of dynamic spectrum functionalities. The radio scene analysis includes frequency and time dependent measurements such as noise floor, traffic data, and the detected spectral holes. The measurements are used to adjust transmit power and operational frequency band. The essence of transmit power control for such a non-cooperative multi-user radio environment can mathematically be stated as following:

For a given and limited number of spectrum holes the transmit power levels of n secondary users is selected in such a manner that data transmission rate of the users is maximised provided interference temperature limit is not violated.

Dynamic spectral management is defined as a modulation strategy that adapts to the time varying conditions of the radio environment. Bandwidth and carrier frequency are dynamically modified according to channel condition. Dynamic spectrum management in case of Orthogonal Frequency Division Multiplexing (OFDM) may entail variation of the number of bits per channel based on the signal-to-noise-ratio (SNR).

1.6.1 Beamforming approach by cognitive radio networks

In cognitive radio environment, it is of prime importance to monitor transmitted power toward primary user. It is essential that power transmitted toward primary user does not exceed interference temperature limit set by primary user themselves. The beamforming techniques are employed as an approach to minimise the transmitted power in the direction of primary user. As such, several beamforming solutions for cognitive radio networks have been proposed in the condition that each CR device is equipped with multiple antennas and the power transmitted toward primary user is minimised through null formation. Transmit power of CR users is preserved by focusing the energy in the direction of the receiver. Therefore, beamforming methods are used to reduce interference to primary user from CR transmission. Due to reciprocity, CR receivers will be able to filter out interference from unwanted transmitter. Spatial variation of the RF stimuli which in this case is primary user transmission is accounted by adaptive beamforming.

One of the key component needed for beamforming is the channel state information (CSI). The base station needs CSI to be aware of the radio environment features - path loss, fading, etc. Precoding in digital systems is done by applying complex weighting

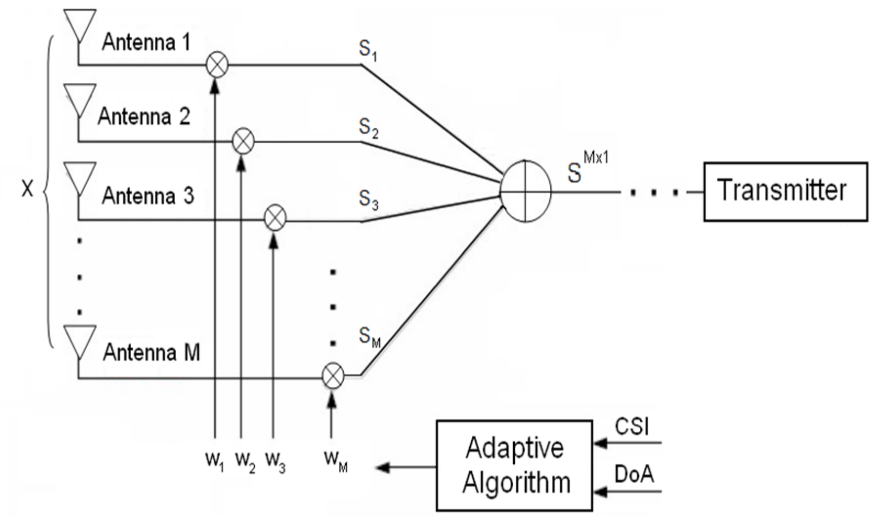

Figure 1.12: Transmit Beamforming.

to the signal before radiating it in the air. Without loss of generality it is assumed that in every antenna element branch, the signal has two components: in-phase (I) and the shifted on 90° quadrature one (Q). The signal in each is scaled by a factor. This is equivalent to multiplication of the complex signal samples (I+jQ) with a set of complex numbers (beamforming weights). Assuming that the signal intended to transmit with M antennas is $s^T \in \mathbb{C}^M$ and the weights are represented with the vector $\mathbf{w} \in \mathbb{C}^M$. Then the resulting output is

$$x = \sum_{i=1}^{M} \mathbf{w}_i s_i. \tag{1.1}$$

This is illustrated in Fig. 1.12. Many different algorithms with a different degree of precision have been used so far for finding the beamforming vectors $\mathbf{w}$. All of them are based on having radio channel knowledge available at the transmitter. A solution that allows maximising SNR at CR receiver and under a transmit power constraint toward primary user is given in a probabilistic context in [28]. The given beamforming solutions are provided considering limited CSI and an upper limit on the transmit power at CR transmitters. As a result, improved (Bit Error Rate) BER at the receiver is obtained while minimising the interference toward CR receiver. Two adaptive beamforming solutions are given in [29] which indicated OFDM as a proper modulation technique for CR. The first beamformer the authors proposed is based on spectrum masking concept which is explained in [30]. This means subcarriers which lie in the same frequency range as primary user are given zero beamforming weight or deactivated. In that way, the the beamforming scheme allowed to suppress interference while maximising the power toward primary user. The second beamforming solution given is based on spatial filtering concept. This was done by steering nulls in the direction of primary user. This allows the primary user and CR to coexist simultaneously.

1.6.2 Downlink beamforming in multi-user cognitive radio networks

Fig. 1.13 illustrates the downlink scenario of a single cell multi-user multiple-input-single-output (MISO) cognitive radio network with multiple secondary users coexisting with multiple primary users. In this configuration, the cognitive radio network is installed far enough from the primary user transmitter (PU-Tx). Although the primary user transmitter is interfering with normal operations of cognitive radio network, the power received from the intended secondary user transmitter (SU-Tx) is much larger, i.e., the interfering power from primary user transmitter can be accumulated as a part of its noise term. Therefore, there are only two sets of relevant CSI which play important roles in the system design. one set describes the channels between secondary user transmitter and secondary users receivers (SU-Rx's) while the other set describes the channels between secondary user transmitter and primary users receivers (PU-Rx). When primary users are inactive, the system becomes a conventional multiuser MISO system, and secondary user link CSI is needed for transmission design. This knowledge is usually acquired through transmitting pilot symbols from secondary user transmitter to secondary users receivers, and feeding back the estimated CSI from secondary users receivers to secondary user transmitter.

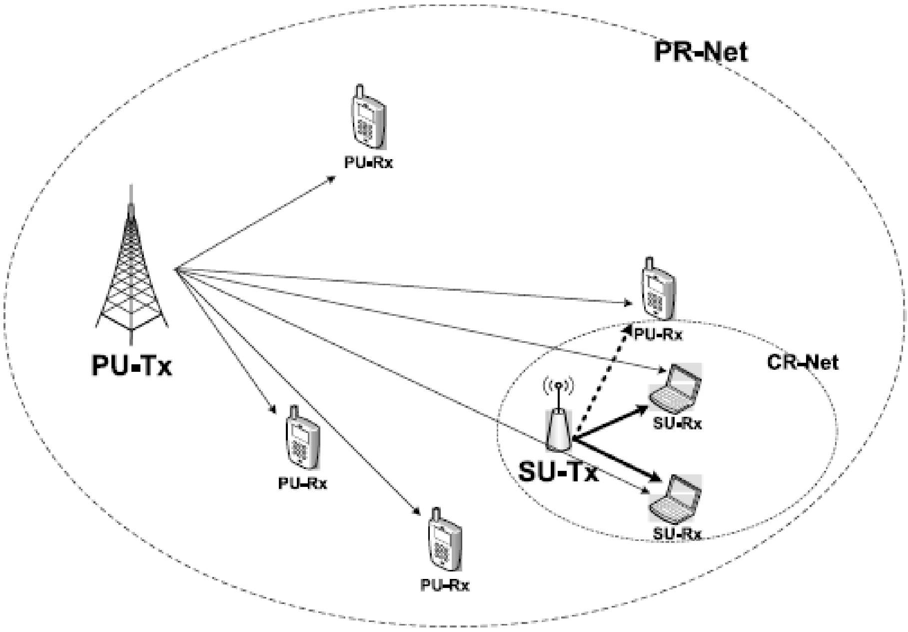

Figure 1.13: Overview of a single-cell cognitive radio network (CR-Net) coexisting with a single cell primary network (PR-Net) [5].

In practice, however, because of the time-varying nature of wireless channels, it is not possible to acquire the CSI perfectly, either due to channel variation and/or channel estimation error and/or feedback error. On the other hand, when primary users are active, primary user link CSI is further needed at secondary user transmitter for the purpose of controlling interferences at the primary users receivers. This CSI knowledge has to be acquired by secondary user transmitter through environmental learning [5], which again may have errors and will be discussed in the following section.

1.7 Robust cognitive beamforming

As mentioned in the previous section, in the design procedure of a communications system, channel gains play an important role. Conventionally it is assumed that the CSI is completely known at the transmit and receive sides. Mostly it is assumed that the receive side may obtain this knowledge through pilot transmission process in which a set of certainly known data is transmitted towards the receiver. As the receiver knows both the transmitted and the received data, it can estimate the channel gain coefficients. Also, it is assumed that there is a perfect feedback channel free of impairments, between the source and the destination of the communication link through which, it is possible to send back the CSI to the transmitter side. Since the beamformer design process relies on this CSI, it is vital to have CSI perfectly. Unfortunately, due to the erroneous channel gain estimation, limited feedback rate between the source and the destination, and rapidly changing environments, this assumption is not a realistic one, and hence, the CSI is uncertain [31].

Nowadays, uncertain (imperfectly known) CSI is modeled using the following notation. If $\hat{H}$ is to represent the real CSI, uncertain CSI, i.e. H, is modeled as

$$H = \hat{H} + e \tag{1.2}$$

where e shows the additive uncertainty of the CSI. To characterize the uncertainty, different models are used, which will be discussed in the following chapter.

It is well known that imperfect CSI can significantly degrade the system performance. In other words, if one derives algorithms for transceiver design based on erroneous channel coefficients as if they were perfect, some promised QoS targets in the system might often be violated. Therefore, designing transmitters and receivers which are robust to imperfect CSI is a task of great practical interest. Robust beamforming [32], [33] is a methodological solution to treat the uncertainty of the CSI.

1.8 Outline of the thesis

The main problem considered in this thesis is the optimisation of multi-user cognitive networks under the assumption of having imperfect/perfect CSI. Depending on a considered scenario, various problems can be defined. The principal contributions are divided in three chapters (Chapters 3, 4, and 5), with respect to assumptions on system setup and channel state information model.

1. INTRODUCTION

This thesis comprises 6 chapters. A brief account of each chapter is given below.

Chapter 1, includes the motivation, overview and states the contributions of the thesis.

Chapter 2, describes an overview of two standard conic programs, i.e. second order cone programming (SOCP) and semidefinite programming (SDP). A linear array antenna used in beamforming is presented followed by the introduction of an optimisation problem of single-cell multiuser beamforming. Algorithms that solve the optimisation problem using SOCP and SDP are recalled. Furthermore, convex and robust optimisation is briefly explained. The presented concepts are used to develop beamforming schemes for a multi-cell cognitive scenario discussed in Chapters 3, 4, and 5.

In chapter 3, the problem of cell-edge user coverage is addressed in a primary cellular network by introducing cognitive cells in the vicinity of primary cell borders. First a cooperative strategy is developed, where the secondary system can benefit from accessing the spectrum of the primary systems when needed to transmit towards secondary users. In return, the secondary base station cooperates with the primary base station by relaying its data towards the primary cell-edge users. Furthermore, a soft interference shaping strategy is introduced, where the interference inflicted on the primary users located outside but within the close vicinity of the cognitive cell borders is controlled within a certain level. The performance of both strategies is evaluated through simulations and presented. These particular contributions are published in [34].

In Chapter 4, an inter-cell interference mitigation method in cognitive cellular network is developed. The method allowed the secondary system to access the spectrum of the primary systems upon need. In return, the primary cell-edge users are supported by the secondary base station, within the cognitive cell, by treating them as secondary users. In addition, the interference towards the primary users (located outside the cognitive cell) caused by the transmission of secondary base station is controlled. The objective of this scheme is to minimise the total transmit power across secondary base stations subject to user SINR constraints within the cognitive cell. The algorithms for finding beamforming vectors are developed based on standard semidefnite programming formulation using instantaneous and second-order-statistical channel state information. The performance of the proposed strategy is compared against the conventional cellular network and discussed in details. The above contributions is submitted and currently under view in [35].

Chapter 5, studies the problem of robust downlink beamforming design in a multiuser MISO cognitive radio network. The first assumption was based on perfect CSI. Furthermore, unlike conventional designs in cognitive radio networks, in this chapter we assume that the CSI for all relevant channels is imperfectly known, and the imperfectness of the CSI is modeled using an Euclidean ball-shaped uncertainty set. The objectives are to minimise the transmit power at the cognitive base station and imposing an upper limit on the Interference at the outer-cell users, subject to targeting a lower bound on the received SINR for all (cell-edge/cognitive) users. The design parameters at the cognitive base station are the beamforming weights. The problem is a nonconvex quadratically constrained quadratic program (QCQP) and in general is hard to achieve the global optimality. As a compromise, using s-procedure based on convex programming, the probabilistic constraints were replaced with conservative deterministic constraints. Finally, simulation results are provided to validate the robustness of the proposed methods. The contributions for this chapter is submitted and currently under view in [36].

The closing Chapter 6 deals with conclusions and future work.

1.9 Publications

A collection of contributions for this thesis has been complied from the following list of publications:

1. T. A. Le, S. Nasseri, A. Zarrebin-Esfahani, A. Mills, and M. R. Nakhai, "Power-efficient downlink transmission in multicell networks with limited wireless back-haul," *IEEE Wireless Communications Magazine, Special Issue on Technologies for Green Radio Communication Networks*, vol. 18, no. 5, pp. 82–88, Oct. 2011.

2. A. Zarrebin-Esfahani, and M. R. Nakhai, "Secondary spectrum access and cell-edge coverage in cognitive cellular networks," *IET Communications, Special Issue on Cognitive Communications*, vol. 6, no. 8, pp. 845-851, May 2012.

3. A. Zarrebin-Esfahani, T. A. Le, and M. R. Nakhai, "Power-Efficient Coverage Scheme for Cell-Edge Users using Cognitive Beamforming," *accepted in IEEE 24th International Symposium on Personal, Indoor and Mobile Radio Communications: Mobile and Wireless Networks*, 8-11 Sept, 2013.

4. A. Zarrebin-Esfahani, T. A. Le, and M. R. Nakhai, "Robust Cognitive Beamforming for Cell-edge Coverage in Multicell Networks with Probabilistic Constraints," *accepted in IEEE GLOBECOM-Cognitive Radio and Networks Symposium*, 9-13 Dec, 2013.

Chapter 2

Mathematical Preliminaries

2.1 Introduction

Having the ability of solving very large, practical engineering problems reliably and efficiently, convex optimisation has become the most widely researched area in optimisation. There is a great race to determine which important problems can be posed in a convex setting. Yet, that skill acquired by understanding the geometry and application of convex optimisation will remain more an art for some time to come; the reason being, there is generally no unique transformation of a given problem to its convex equivalent. This means, two researchers pondering the same problem are likely to formulate the equivalent differently, hence, one solution is likely different from the other for the same problem. Any presumption of only one right or correct solution becomes nebulous. Since a local minimum in a convex optimisation problem is also the global minimum, the global minimum can be attained by any "Gradient Descent" or "Hill Climbing" algorithm [6]. Linear programming, i.e., a program with linear objective function and linear/affine constraints, is a well researched topic in convex programming. Recent developments in convex programming extend the results and algorithms of linear programming to more complicated convex programs, e.g., conic programming. A conic programming is a linear programming with generalised inequalities.

This chapter concisely reviews linear programming, convex optimisation, and robust optimisation. Two standard conic programs, i.e. second order cone programming (SOCP) and semidefinite programming (SDP) are reviewed. Concepts of a linear antenna array used for beamforming are described. Applications of SOCP and SDP for solving the problem of multiuser beamforming in a single-cell scenario are discussed. The concepts presented in this chapter are beneficial to the developments of beamforming schemes introduced in Chapters 3, 4, and 5. Readers interested in convex optimisation and applications of convex optimisation in communications are referred to [6], [37], and [38] for more details.

2.2 Convex optimisation

A convex program is an optimisation problem where we seek the minimum of a convex function over a convex set. Its objective function as well as the constraints are convex. Convex optimisation problems often occur in signal processing, communications, structural analysis and many other fields. Convex problems can be solved numerically with great efficiency and global optimums can be obtained. Efficient interior-point methods are available for the solution of convex optimisation problems. However, the difficulty is often to recognize convexity; convexity is harder to recognize than say, linearity. One important feature of convexity is that it is possible to address difficult, nonconvex problems (such as combinatorial optimisation problems) using convex approximations that are more efficient than classical linear ones. Convex optimisation is especially relevant when the data of the problem at hand is uncertain, and robust solutions are sought.

2.2.1 Convex set

A set C is convex if the line segment between any two points in C lies in C, i.e., if for any x_1, $x_2 \in C$ and any θ with $0 \leq \theta \leq 1$, we have

$$\theta x_1 + (1 - \theta)x_2 \in C. \tag{2.1}$$

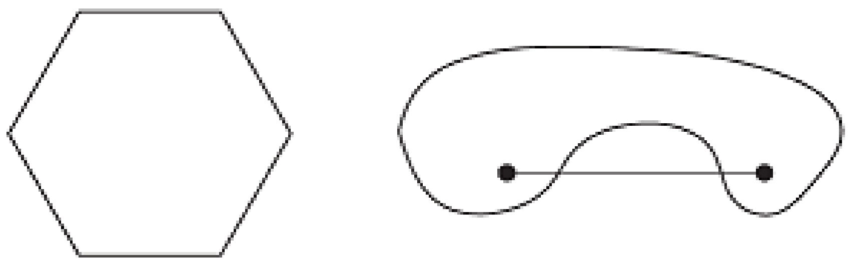

Figure 2.1: Some simple convex and nonconvex sets [6].

In other words, in a convex set C every point in the set can be seen by every other point, along an unobstructed straight path between them, where unobstructed means lying in the set. Some simple convex and nonconvex sets is shown in Fig. 2.1. The hexagon including its boundary is a convex set whereas the kidney shaped set is not convex, since the line segment between the two points is partly not contained in the set.

2.2.2 Convex cone

A set C is called a Cone, if for every $x \in C$ and $\theta \geq 0$ we have $\theta x \in C$. The set C is called a convex cone if it is convex and a cone, which means that for any x_1, $x_2 \in C$ and $\theta_1, \theta_2 \geq 0$, we have

$$\theta_1 x_1 + \theta_2 x_2 \in C. \tag{2.2}$$

Points of this form can be described geometrically as forming the two-dimensional 'pie-slice', with apex 0 and edges passing through x_1 and x_2 as shown in Fig. 2.2. The pie-slice shows all the points of the form $\theta_1 x_1 + \theta_2 x_2$, where $\theta_1, \theta_2 \geq 0$. The apex of the slice that corresponds to $\theta_1 = \theta_2 = 0$ is at 0 and its edges pass through the points x_1 and x_2.

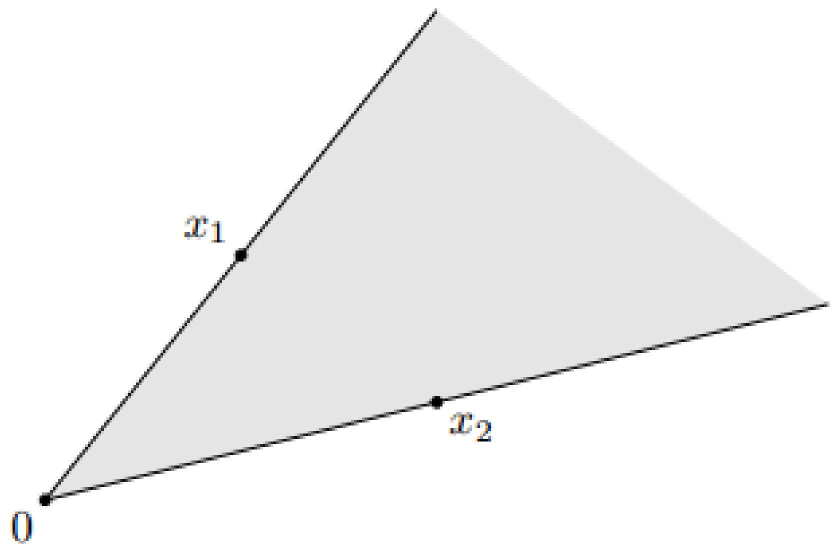

Figure 2.2: Example of a convex cone [6].

2.2.3 Convex functions

A function $f : \mathbb{R}^n \to \mathbb{R}$ is convex if the domain of f is convex and for all x, y that belong to the domain of f and for any $0 \leq \theta \leq 1$, we have

$$f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta)f(y). \tag{2.3}$$

The function f is strictly convex if strict inequality holds in 2.3 whenever $x \neq y$ and $0 < \theta < 1$. It can be said that f is concave, if $-f$ is convex, and strictly concave if $-f$ is strictly convex. Geometrically, the inequality 2.3 can be interpreted as a line segment between $(x, f(x))$ and $(y, f(y))$ that lies above the graph of f as shown in Fig 2.3. Some of the examples of convex functions are

- Exponential; e^{ax} is convex on $\mathbb{R}$, for any $a \in \mathbb{R}$.
- Powers of absolute value; $|x|^p$, for $p \geq 1$, is convex on $\mathbb{R}$.
- Powers; x^a, for $a \geq 1$ or $a \leq 0$, is convex on $\mathbb{R}_{++}$.

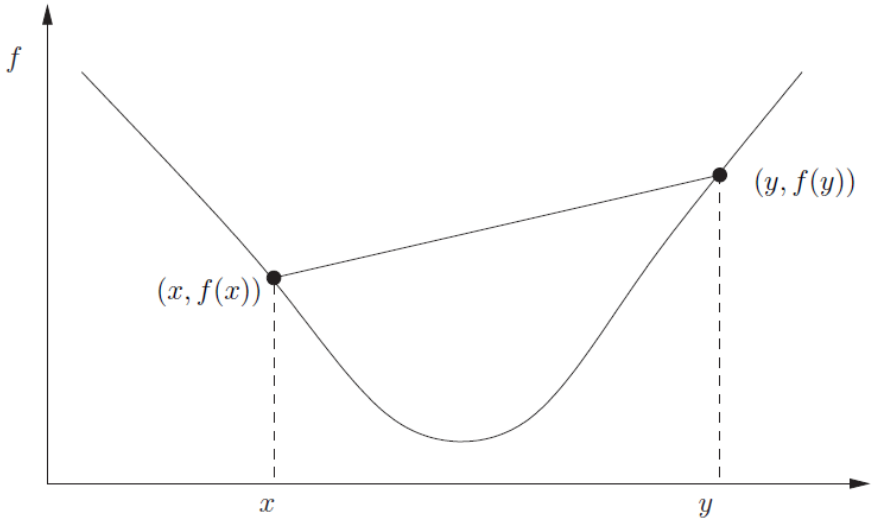

Figure 2.3: Graph of a convex function [6].

2.2.4 Convex optimisation problem

The generic standard form of an optimisation problem [39] is as follows

$$\begin{aligned} \min_{\mathbf{x}} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \leq 0, \quad i = 1, 2, .., m, \\ & h_i(\mathbf{x}) = 0, \quad i = 1, 2, .., p, \end{aligned} \tag{2.4}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the optimisation variable, f_0: $\mathbb{R}^n \to \mathbb{R}$ is the objective function, f_i: $\mathbb{R}^n \to \mathbb{R}$, $i = 1, 2, .., m$, are the inequality constraint functions, and h_i: $\mathbb{R}^n \to \mathbb{R}$, $i = 1, 2, .., p$ are the equality constraint functions. This notation is to describe a problem which tries to find the minimum value of the objective function $f_0(\mathbf{x})$ subject to m and p inequality and equality constraints, respectively.

In its standard form, a convex optimisation problem can be expressed as

$$\begin{aligned} \min_{\mathbf{x}} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \leq 0, \quad i = 1, 2, .., m, \\ & \mathbf{a}_i^T \mathbf{x} = b_i, \quad i = 1, 2, .., p. \end{aligned} \tag{2.5}$$

where $f_0, f_1, \ldots, f_m$ are convex and $\mathbf{a}_i$ and b_i, $i = 1, 2, .., p$, are fixed parameters.

The convexity is often considered as a criterion that separates efficiently solvable from difficult optimisation problems. Almost all convex problems can be solved, either in a closed form or using iterative algorithms. Some classes of them have very efficient numerical solutions.

2.2.5 Linear programming

The most commonly used convex optimisation problem is the Linear Programming (LP) problem, an optimisation problem with linear objective and linear inequality constraints:

$$\begin{aligned} &\min && \mathbf{f}^T\mathbf{x} \\ &\text{subject to} && \mathbf{a}_i^T\mathbf{x} \leq b_i, \quad i = 1, 2, .., p. \end{aligned} \tag{2.6}$$

where the optimisation variable is the vector $\mathbf{x}$, and $\mathbf{a}_i \in \mathbb{R}^n$, $b_i \in \mathbb{R}$, and $\mathbf{f} \in \mathbb{R}^n$ are the problem parameters. Solving linear programs are reliable and computation time proportional to n^2p if $p \geq n$.

More special cases of convex optimisation problems that are mostly used in science and technology, namely, Second Order Cone Programming (SOCP) problems and Semidefinite Programming (SDP) problems. In this thesis we mostly focus on SOCP and SDPs, as the objective and constraints of beamforming design problems are stated using these problems.

2.3 Second order cone programming

Suppose $\|.\|_l$ is any norm on $\mathbb{R}^n$. From the general properties of the norms, it can be shown that a norm ball of radius r with center $\mathbf{x}_c$, given by $\{\mathbf{x}|, \|\mathbf{x} - \mathbf{x}_c\|_l < r\}$, is convex.

The cone associated with the norm $\|.\|_l$ is

$$C = \{(\mathbf{x}, t)|, \|\mathbf{x}\|_l < t\}. \tag{2.7}$$

The SOC is the norm cone for the Euclidean norm ($l = 2$) and is described as

$$C = \left\{(\mathbf{x}, t) \in \mathbb{R}^{n+1} |, \|\mathbf{x}\|_2 < t \right\|. \tag{2.8}$$

where $n+1$ is also known as the dimension of the cone. A convex optimisation problem with SOC constraints is also known as a SOCP problem, which in general has the following form [6], [40]:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{f}^T\mathbf{x} \\ \text{subject to} \quad & \|\mathbf{A}_i\mathbf{x} + \mathbf{b}_i\|_2 \leq \mathbf{c}_i^T\mathbf{x} + d_i, \quad \forall 1 \leq i \leq N, \end{aligned} \tag{2.9}$$

where $\|.\|_2$ denotes the Euclidean norm, i.e, for any $\mathbf{z} \in \mathbb{R}^n$, $\|\mathbf{z}\|_2 = \sqrt{\mathbf{z}^T\mathbf{z}}$, the vector $\mathbf{x}$ is the optimisation variable with the length of n; $\mathbf{f}$, $\mathbf{A}_i$, $\mathbf{b}_i$, $\mathbf{c}_i$ and d_i are deterministic parameters with appropriate sizes.

Another interesting special case arises when $\mathbf{c}_i = 0$, so the i^{th} SOC constraint reduces to $\|\mathbf{A}_i\mathbf{x} + \mathbf{b}_i\|_2 \leq d_i$, which is equivalent (assuming $d_i \geq 0$) to the (convex) quadratic constraint $\|\mathbf{A}_i\mathbf{x} + \mathbf{b}_i\|^2 \leq d_i^2$. Thus, when all d_i vanish, the SOCP problem reduces to a Quadratically Constrained Linear Program (QCLP). The (convex) Quadratic Programs (QPs), Quadratically Constrained Quadratic Programs (QCQPs), and many other nonlinear convex optimisation problems can be reformulated as SOCP problems as well. Thus SOCP problems include LP problems and QPs as special cases, but can also be used to solve a variety of nonlinear, non differentiable problems.

2.4 Semidefinite programming

Let us consider a set of Hermitian $n \times n$ matrices represented by $\mathbf{S}^n$ as

$$\mathbf{S}^n = \left\{\mathbf{X} \in \mathbb{C}^{n\times n} | \mathbf{X} = \mathbf{X}^H\right\}, \tag{2.10}$$

which is a vector space with dimension $n(n+1)$. Using the notation $\mathbf{S}^n{}_+$, the set of Hermitian positive semidefinite matrices can be represented as

$$\mathbf{S}^n{}_+ = \{\mathbf{X} \in \mathbf{S}^n | \mathbf{X} \succeq 0\}, \tag{2.11}$$

where the notation $\mathbf{X} \succeq 0$ represents that the matrix $\mathbf{X}$ is positive semidefinite.

SDP unifies several standard problems (e.g., linear and quadratic programming) and finds many applications in engineering and combinatorial optimisation. Although SDP problems are much more general than LP problems, they are not much harder to solve. Most interior-point methods for LP have been generalized to SDP problems. As in LP, these methods have polynomial worst-case complexity, and perform very well in practice. The standard form of a SDP is defined as [6], [40]:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{f}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{A}(\mathbf{x}) \succeq 0, \end{aligned} \tag{2.12}$$

where

$$\mathbf{A} = \mathbf{A}_0 + \sum_{i=1}^{m} x_i \mathbf{A}_i \tag{2.13}$$

is a Hermitian matrix that depends affinely on $\mathbf{x}$ and the $n \times n$ Hermitian matrix $\mathbf{A}_i$ is deterministic data. In general, the dual problem associated with the SDP (2.12) is [41]

$$\begin{aligned} \max_{\mathbf{X}} \quad & -\operatorname{Tr}(\mathbf{A}_0\mathbf{X}) \\ \text{subject to} \quad & \operatorname{Tr}(\mathbf{A}_i\mathbf{X}) = b_i, \quad i = 0, \ldots, m, \\ & \mathbf{X} = \mathbf{X}^H \succeq 0, \end{aligned} \tag{2.14}$$

where $\mathbf{F}_0$, $\mathbf{A}_l$, and $\mathbf{X}$ are all $n \times n$ symmetric matrices, b_i is a scalar and the constraint, and $\mathbf{X}^H \succeq 0$ denotes that the matrix $\mathbf{X}$ is a Hermitian positive semidefinite. Even this is a highly nonlinear constraint, it is still convex because a set of positive semidefinite matrices form a convex cone.

The dual problem (2.14) is also a SDP like the primal problem, i.e., it can be cast in the same form as the primal problem (2.12). The proof [41] is sketched in the following.

For simplicity, assuming that the matrices $\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_m$ are linearly independent. Then the affine set $\text{Tr}(\mathbf{A}_i\mathbf{X}) = b_i, \forall i$ can be expressed in the form:

$$\mathbf{G}(\mathbf{y}) = \mathbf{G}_0 + y_1\mathbf{G}_1 + \cdots + y_p\mathbf{G}_p \tag{2.15}$$

where $p = n(n+1)/2 - m$ and $\mathbf{G}_i$ are appropriate matrices. Defining

$$\mathbf{d} = \begin{bmatrix}\text{Tr}(\mathbf{F}_0\mathbf{G}_1) & \text{Tr}(\mathbf{F}_0\mathbf{G}_2) & \cdots & \text{Tr}(\mathbf{F}_0\mathbf{G}_p)\end{bmatrix}^T.$$

Hence,

$$\mathbf{d}^T\mathbf{y} = \text{Tr}\left(\mathbf{F}_0[\mathbf{G}(\mathbf{y}) - \mathbf{G}_0]\right).$$

Therefore, the dual problem (2.14) becomes

$$\begin{aligned} \min_{\mathbf{y}} \quad & \mathbf{d}^T\mathbf{y} \\ \text{subject to} \quad & \mathbf{G}(\mathbf{y}) \succeq 0 \end{aligned} \tag{2.16}$$

which is a standard SDP form defined in (2.12). This concludes that the problem (2.14) is also a SDP.

Many convex optimisation problems, e.g., LP and (convex) QCQPs, can be cast as SDP problems, so that SDP offers a unified way to study the properties and derive algorithms for a wide variety of convex optimisation problems. SDP problems include LP and SOCP problems as special cases, but can also be used to solve many other nonlinear, non differentiable problems.

In the following, a transformation from a second-order-cone constraint to a semidefinite constraint, also known as a linear matrix inequality (LMI) constraint, is presented. Defining a matrix $\mathbf{M} \in \mathbb{C}^{(q+t)\times(q+t)}$:

$$\mathbf{M} = \begin{bmatrix}\mathbf{A}_{q\times q} & \mathbf{B}_{q\times t} \\ \mathbf{C}_{t\times q} & \mathbf{D}_{t\times t}\end{bmatrix} \tag{2.17}$$

where the dimensions of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ as shown in the block display, $\mathbf{A}$ and $\mathbf{D}$ are square matrices but $\mathbf{B}$ and $\mathbf{C}$ are not square unless $t = q$. Recall the Schur complement definition:

Definition 2.1. *If* $\mathbf{D}$ *is nonsingular, the Schur complement of* $\mathbf{M}$ *with respect to* $\mathbf{D}$ *is defined as*

$$\mathbf{S} = \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}. \tag{2.18}$$

Matrix $\mathbf{S}$ has the following main properties [37]:

- $\mathbf{M} \succ 0$ if and only if $\mathbf{D} \succ 0$ and $\mathbf{S} \succ 0$.
- If $\mathbf{D} \succ 0$, then $\mathbf{M} \succeq 0$ if and only if $\mathbf{S} \succeq 0$.

Since $\mathbf{c}_i^H\mathbf{x} + d_i > 0$, $\left(\mathbf{c}_i^H\mathbf{x} + d_i\right)\mathbf{I} \succ 0$. Using the second property of $\mathbf{S}$, one can show that the ith SOC constraint in (2.9), i.e.,

$$\|\mathbf{A}_i^H\mathbf{x} + \mathbf{b}_i\| \leq \mathbf{c}_i^H\mathbf{x} + d_i, \tag{2.19}$$

is equivalent to the following LMI constraint:

$$\begin{bmatrix} \mathbf{c}_i^H\mathbf{x} + d_i & \mathbf{x}^H\mathbf{A}_i + \mathbf{b}_i^H \\ \mathbf{A}_i^H\mathbf{x} + \mathbf{b}_i & \left(\mathbf{c}_i^H\mathbf{x} + d_i\right)\mathbf{I} \end{bmatrix} \succeq 0. \tag{2.20}$$

Hence (2.9) can be written in a standard SDP form as:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{f}^T\mathbf{x} \\ \text{subject to} \quad & \begin{bmatrix} \mathbf{c}_i^H\mathbf{x} + d_i & \mathbf{x}^H\mathbf{A}_i + \mathbf{b}_i^H \\ \mathbf{A}_i^H\mathbf{x} + \mathbf{b}_i & \left(\mathbf{c}_i^H\mathbf{x} + d_i\right)\mathbf{I} \end{bmatrix} \succeq 0, \quad \forall 1 \leq i \leq N. \end{aligned} \tag{2.21}$$

The SeDuMi solver [42] is a common optimisation packet that can be used to solve SOCP and SDP. An elegant Matlab-based modeling system for convex optimisation, i.e., CVX which supports the SeDuMi solver, has been developed by Michael Grant and Stephen Boyd [43].

2.5 Robust optimisation

This section provides some points that are of particular interest when modeling uncertain parameters in robust optimisation problems [44]. Important applications have already been found in operations research, estimation, and control theory. In this thesis, we will be use this for designing wireless communication systems, robust to imperfect knowledge of the channel.

2.5.1 Uncertainty problem

The parameters in optimisation problems are often uncertain. Recall the LP problem

$$\begin{aligned} &\min && \mathbf{f}^T\mathbf{x} \\ &\text{subject to} && \mathbf{a}_i^T\mathbf{x} \leq b_i, \quad i = 1, 2, .., p. \end{aligned} \tag{2.22}$$

In practice, however there exists uncertainty including

- Errors in data $(\mathbf{f}, \mathbf{a}_i, b_i)$,
- Errors in implementation of the solution. It can be either absolute error: $\mathbf{x}^* \to \mathbf{x}^* + \delta$, for small δ, or relative error: $\mathbf{x}^* \to (1 + \Delta)\mathbf{x}^*$, for diagonal and small Δ.

Implementation errors can be regarded as a particular case for data error as

- Absolute error: The constraint becomes $\mathbf{a}_i^T(\mathbf{x}+\delta) \leq b_i$, i.e., $\mathbf{a}_i^T\mathbf{x} \leq b_i - \mathbf{a}_i^T\delta = b(\delta)$,
- Relative error: The constraint becomes $\mathbf{a}_i^T(1+\Delta)\mathbf{x} \leq b_i$, i.e., $\mathbf{a}_i^T(\Delta) = \mathbf{a}_i^T(1+\Delta)$.

Sometimes the uncertainty has a significant effect on the optimal solution and optimal value of the problem.

2.5.2 Uncertainty model

Suppose that the data in LP is $(\mathbf{f}, \mathbf{a}_i, b_i)$, and $\delta \in \mathbb{R}^n$. A uncertainty model is defined by

- The map $\delta \to (\mathbf{f}(\delta), \mathbf{a}_i(\delta), b_i(\delta))$, and
- Bounds on δ.

2.5.3 Robust optimisation framework

There are different models to describe this uncertainty. In LP, the uncertainty model is $(\mathbf{f}, \mathbf{a}_i, b_i) \in \mathcal{U}$, where $\mathcal{U} \in \mathbb{R}^n$ is the uncertainty set which, for our purposes, will always be closed.. Assume $\mathbf{f}$ is a constant, the robust LP problem is shown as

$$\begin{aligned} \min \quad & \mathbf{f}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{a}_i^T\mathbf{x} \leq b_i, \quad \forall(\mathbf{a}_i, b_i \in \mathcal{U}). \end{aligned} \tag{2.23}$$

If $\mathbf{f}$ is also uncertain, the problem can be written as

$$\begin{aligned} \min_{\mathbf{x},t} \quad & t \\ \text{subject to} \quad & t \geq \mathbf{f}^T\mathbf{x}, \\ & \mathbf{a}_i^T\mathbf{x} \leq b_i, \quad \forall(\mathbf{a}_i, b_i, \mathbf{f} \in \mathcal{U}). \end{aligned} \tag{2.24}$$

Robust conic program can be shown as

$$\begin{aligned} \min_{\mathbf{x},t} \quad & t \\ \text{subject to} \quad & t \geq \mathbf{f}^T\mathbf{x}, \\ & b_i - \mathbf{a}_i^T\mathbf{x} \in \mathcal{K}, \quad \forall(\mathbf{a}_i, b_i, \mathbf{f} \in \mathcal{U}). \end{aligned} \tag{2.25}$$

Finally robust convex program [45] is formulated as

$$\begin{aligned} \min_{\mathbf{x}} \quad & f_0(\mathbf{x}, \delta) \\ \text{subject to} \quad & f_i(\mathbf{x}, \delta) \leq 0, \quad \forall\delta \in \mathcal{U}. \end{aligned} \tag{2.26}$$

The above robust convex program is convex if for any δ, $f_i(\mathbf{x}, \delta)$ is convex in $\mathbf{x}$, for $i = 1, 2, .., p$. The objective and constraints must be satisfied for any occurrences of the data in the uncertainty set $\mathcal{U}$. Since there are infinite number of realisations for this data, the uncertain optimisation problems are called semi-infinite problems. As it is known, these problems are hard to solve, and therefore, three different common approaches are considered to relax the semi-infinite problems in this area. In each approach, a known mechanism is exploited to reduce the number of constraints. These approaches are namely:

- Stochastic model [46], [47], where the optimisation is to optimise the average (mathematical expectation) of the objective and constraints. In this case, the robust counterpart of the uncertain problem would be

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{E}_\delta[f_0(\mathbf{x},\delta)] \\ \text{subject to} \quad & \mathbf{E}_\delta\left[f_i(\mathbf{x},\delta)\right] \leq 0, \quad \forall \delta \in \mathcal{U}. \end{aligned} \tag{2.27}$$

- Chance model, where chance constraints are a probabilistic way of handling probabilistic uncertainty. Let us consider the following optimisation problem

$$\begin{aligned} \min_{\delta \in \mathcal{U}} \quad & f_0(\mathbf{x},\delta) \\ \text{subject to} \quad & \Pr\left\{f_i(\mathbf{x},\delta) \leq 0\right\} \geq 1-\rho, \quad \forall \delta \in \mathcal{U}, \end{aligned} \tag{2.28}$$

  where $\Pr\{A\}$ denotes probability of an event A, and $\rho \in (0,1]$ is a preselected value, making sure that the inequality constraints are no greater than a threshold at least $(1-\rho)\times 100\%$ of the time. Probability constraints of the form appearing in (2.28) arise naturally in various applications and are called chance (or probabilistic) constraints. Such constraints can be viewed as a compromise with the requirement of enforcing the constraints $f_i(\mathbf{x},\delta) \leq 0$ for all values $\delta \in \mathcal{U}$ of the uncertain data vector, which could be too costly or even impossible. Chance constrained optimisation problems were introduced in [48], [49].

The probabilistic constraints are usually nonconvex and mathematically intractable (no closed-form expression) in general, therefore, approximation methods are considered. We present two approximation methods based on Relax-and-Restrict idea

1. Relax: Semidefinite relaxation, where we relax the nonconvex quadratic terms to linear terms. The relaxed problem is simpler since the argument in the probability functions are linear. However, the probability constraints are still intractable. Therefore, we seek restrictive (conservative) approximations, in order to guarantee the satisfaction of the probability constraints.

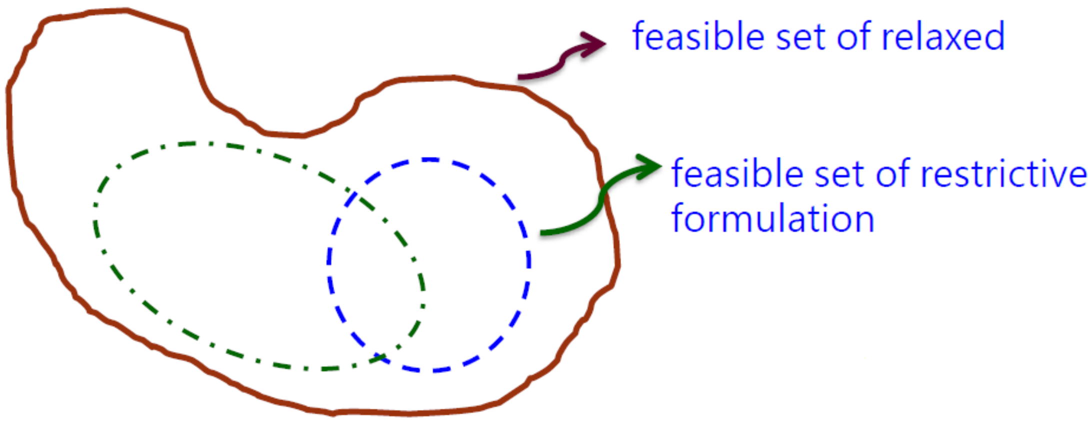

Figure 2.4: Feasible sets of relaxed and subsets.

2. Restrict: Restrictive convex approximations to probability constraints, where the idea of restrictive approximation [50] is to find a set of constraints which corresponds to a feasible set that is convex and is a subset of the feasible set of the relaxed as depicted in Fig. 2.4

- Worst-case model, where the objective and constraint functions are replaced with their least favorable representations [51], [52]

$$\begin{aligned} \min_{\mathbf{x}} \quad & \max_{\delta \in \mathfrak{U}} f_0(\mathbf{x}, \delta) \\ \text{subject to} \quad & \max_{\delta \in \mathfrak{U}} f_i(\mathbf{x}, \delta) \leq 0, \quad i = 1, \ldots, m. \end{aligned} \tag{2.29}$$

It is generally understood that starting from a SOCP problem with uncertain data, would lead to a SDP. To convert the original SOCP to its SDP representation the S-Procedure [43] is frequently used. Therefore, the infinitely many constraints can be recast as a finite number of linear matrix inequalities.

One of the fundamental results in robust optimisation theory is the S-Procedure [43].

Lemma 2.1 (The S-Procedure)**.** *Let*

$$f_i(\mathbf{x}_i) = \mathbf{x}_i^H \mathbf{A}_i \mathbf{x}_i + 2Re\left\{\mathbf{x}_i^H \mathbf{b}_i\right\} + c_i, \quad \textit{for } i = 1, 2, \tag{2.30}$$

where $\mathbf{A}_i$ is $n \times n$ symmetric matrix, $\mathbf{b}_i \in \mathbb{R}^n$, and $c_i \in \mathbb{R}$. Suppose that there exists an $\hat{\mathbf{x}}_i \in \mathbb{C}^M$ such that $f_2(\hat{\mathbf{x}}_i) < 0$. Then for all $\mathbf{x}_i \in \mathbb{C}^M$, the following two conditions are equivalent

1. *$f_1(\mathbf{x}_i) \geq 0$ and $f_2(\mathbf{x}_i) \leq 0$ are satisfied for all $\mathbf{x}_i$,*

2. *There exists a $\lambda \geq 0$ such that*

$$\begin{bmatrix} \mathbf{A}_1 + \lambda \mathbf{A}_2 & \mathbf{b}_1 + \lambda \mathbf{b}_2 \\ \mathbf{b}_1^H + \lambda \mathbf{b}_2^H & c_1 + \lambda c_2 \end{bmatrix} \succeq 0.$$

2.6 Introduction to multiple antenna systems

A multi-antenna communication system [7] is shown in Fig. 2.5. A binary data stream from a compressed digital source is fed to a transmitter which in general introduces error control coding and mapping to complex modulation symbols (QPSK, M-QAM, etc). Several separate symbol streams produced by the modulator are then mapped onto the multiple transmit antennas. Linear spatial weighting of the antenna signals or linear antenna space-time precoding can be used for mapping. After upward frequency conversion, filtering, and amplification, the signals are transmitted through a wireless channel. At the receiver, the signals are received by multiple antennas followed by the demodulation and demapping operations to recover the message. The selection of coding and antenna mapping algorithms depends on different factors like the availability of CSI, complexity and the application type. This determines the class and performance of the multi-antenna system that is implemented. As subscriber units are gradually evolving to become sophisticated wireless Internet access devices rather than just pocket telephones, the stringent size and complexity constraints are becoming somewhat more relaxed. This makes multiple antenna transceivers a possibility at both sides of the link, although from the engineering point of view it is more logical to push most of the processing and cost to the network side like the base station or base station controller.

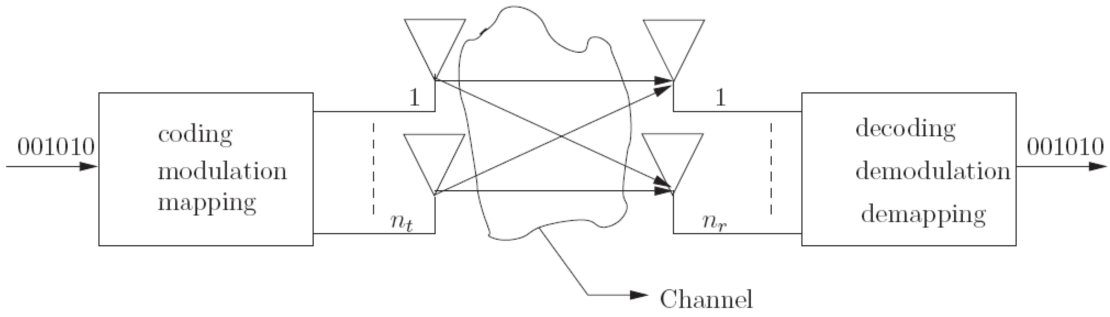

Figure 2.5: Multi-antenna wireless communication system [7].

2.7 Linear antenna array

Smart antennas are composed of two or more antennas working in harmony to create a unique radiation pattern for the electromagnetic environment at hand. The antenna elements are allowed to work in harmony by means of the array element phasing, which is performed with hardware or is carried out digitally [53]. Arrays of antennas can be in any geometry form such as linear arrays, circular arrays, planar arrays, and conformal arrays. In this section, a concept of a linear antenna array in [54] is reviewed. Thorough treatments for all arrays of antennas can be found in [55] and [56]. Consider a signal wavefront, $z(t)$, impinging on an antenna array comprising M antennas spaced d apart each other at angle θ, shown in Fig. 2.6. It is assumed that the wavefront has a bandwidth B and is expressed as:

$$z(t) = \beta(t)e^{j2\pi\nu_c t} \tag{2.31}$$

where $\beta(t)$ is the complex envelope representation of the signal and ν_c is the carrier frequency. Let T_z be the traveling time of the wavefront across any two adjacent antennas. It is clear that

$$T_z = \frac{dsin(\theta)}{c} \tag{2.32}$$

where c is the speed of light.

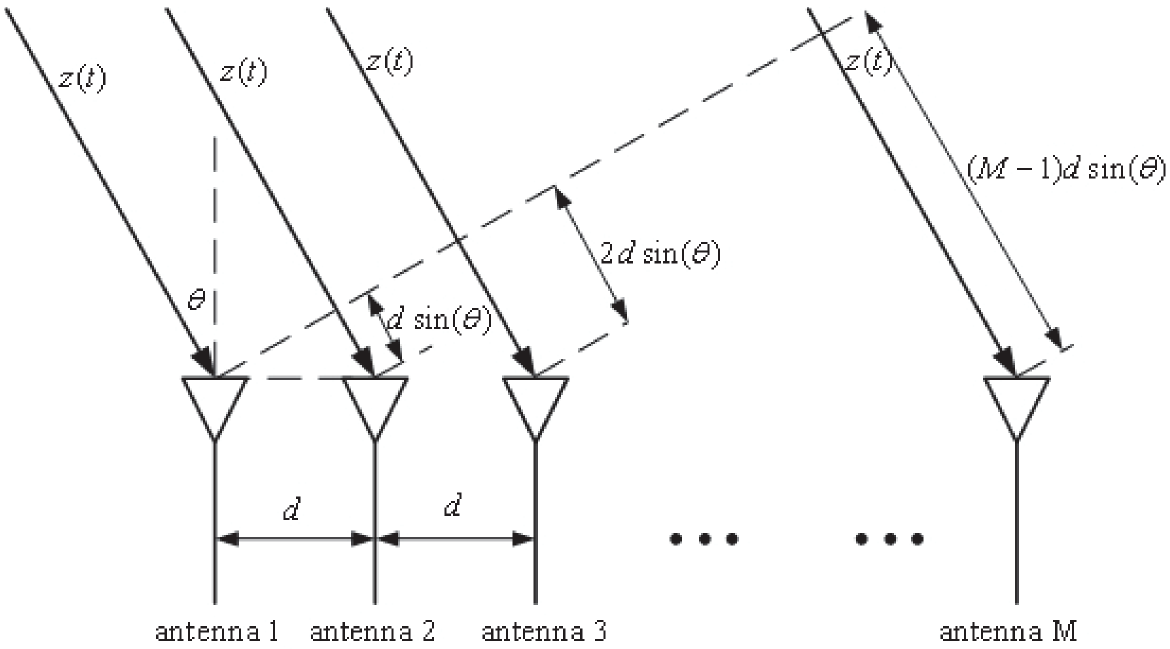

Figure 2.6: Schematic of a wavefront impinging across an antenna array. Under the narrowband assumption the antenna outputs are identical except for a complex scalar [8].

The maximum time of the wavefront traveling along one array is assumed to be much smaller than the reciprocal of the bandwidth of all transmitted signals, i.e.,

$$B \ll \frac{1}{(M-1)T_z}. \tag{2.33}$$

Assuming that antenna element patterns are identical. Provided the received signal at the first antenna is

$$y_1(t) = z(t) = \beta(t)e^{j2\pi\nu_c t}, \tag{2.34}$$

then the received signal at the second antenna is

$$y_2(t) = z(t-T_z) = \beta(t-T_z)e^{j2\pi\nu_c(t-T_z)}. \tag{2.35}$$

Under the narrowband assumption in (2.33), $B \ll 1/T_z$. It can be stated that

$$\beta(t-T_z) \approx \beta(t). \tag{2.36}$$

Let λ_c be the wavelength of the signal wavefront. Using $\nu_c/c = 1/\lambda_c$, (2.32), (2.34) and (2.36) in (2.37), one can arrive at

$$y_2(t) = y_1(t)e^{-j2\pi sin(\theta)\frac{d}{\lambda_c}}. \tag{2.37}$$

Similarly, the received signal at the kth antenna, i.e., $k = 1, 2, \cdots, M$, is given as

$$y_k(t) = y_1(t)e^{-j2\pi(k-1)sin(\theta)\frac{d}{\lambda_c}}. \tag{2.38}$$

From (2.34), (2.37) and (2.38), it can be seen that the signals received at any two array elements are identical except for a phase shift which depends on the angle of arrival and the array geometry.

Consider a free field environment, i.e., neither scatterers and nor multipath. A planar continuous-wave wavefront of frequency ν_c arriving from an angle θ will introduce a spatial signature across the antenna array. This spatial signature is a function of angle of arrival, antenna element patterns and antenna array geometry. The complex $M \times 1$ vector, $\mathbf{a}(\theta) = \begin{bmatrix} a_1(\theta) & a_2(\theta) & \cdots & a_M(\theta) \end{bmatrix}^T$, is called the array response vector. For the linear antenna array with identical element patterns, the array response vector is given as

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ e^{-j2\pi sin(\theta)\frac{d}{\lambda_c}} \\ \vdots \\ e^{-j2\pi(M-1)sin(\theta)\frac{d}{\lambda_c}} \end{bmatrix}. \tag{2.39}$$

Similarly, it is possible write the array response vector for a transmit linear antenna array with identical element patterns as

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 & e^{-j2\pi sin(\theta)\frac{d}{\lambda_c}} & \cdots & e^{-j2\pi(M-1)sin(\theta)\frac{d}{\lambda_c}} \end{bmatrix}. \tag{2.40}$$

Hence, the MISO channel between the antenna array and a user i can be written as

$$\mathbf{h}_i = \xi_i \mathbf{a}(\theta_i) \tag{2.41}$$

where ξ_i captures both effects of channel fading, i.e. fast and slow fading, and pathloss, θ_i is the angle of departure, with respect to the broadside of the antenna array, of the user i.

Using antenna arrays opens up a spatial dimension to improve capacities of wireless communication systems. This improvement is due to the fact that smart beam patterns can be shaped by controlling the phases of individual antennas of the array. Hence power-efficient beams can be steered towards intended users while minimum/non interference are imposed on unintended users. Smart beam patterns are performed via algorithms based on certain criteria. These algorithms can be implemented using hardware. However, it is more easily performed using software, i.e., using digital signal processing [53]. These criteria could be either minimising transmit power with constraints on users' SINRs or maximising users' sum rate with constraints on transmit power to name a few. In the following section, the first strategy, i.e., minimising transmit power under constraint of users' SINR, is reviewed.

2.8 Multiuser downlink beamforming

Consider a base station (BS) equipped with an array of M antenna elements transmitting to U single-antenna users. The signal received by any user i, i.e., $y_i, i \in \{1, \cdots, U\}$, is given by

$$y_i = \mathbf{h}_i \mathbf{w}_i s_i + \sum_{j=1, j \neq i}^{U} \mathbf{h}_i \mathbf{w}_j s_j + n_i \tag{2.42}$$

where $\mathbf{h}_i \in \mathcal{C}^{1 \times M}$ is the MISO vector channel between user i and the BS, $\mathbf{w}_i \in \mathcal{C}^{M \times 1}$ represents the beamforming vector for user i, s_i is the intended symbol for user i and finally n_i is the zero mean circularly symmetric complex Gaussian (ZMCSCG) random variable, i.e., $n_i \sim \mathcal{N}(0, \sigma^2)$, modeling the additive white Gaussian noise at the receiving point of user i. Without loss of generality, assuming that $\mathbb{E}\left(|s_i|^2\right) = 1, \forall i$.

The signal-to-interference-plus-noise ratio for any user i is expressed as

$$\mathrm{SINR}_i = \frac{|\mathbf{h}_i \mathbf{w}_i|^2}{\sum_{j=1, j\neq i}^{U} |\mathbf{h}_i \mathbf{w}_j|^2 + \sigma^2}. \tag{2.43}$$

A common class of optimal transmit downlink beamforming for multiple users is to find a set of $\mathbf{w}_i$ that minimises the total transmit power while guaranteing all users' SINR requirements $\gamma_i, \forall i$:

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & \sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{w}_i \\ \text{subject to} \quad & \frac{|\mathbf{h}_i \mathbf{w}_i|^2}{\sum_{j=1, j\neq i}^{U} |\mathbf{h}_i \mathbf{w}_j|^2 + \sigma^2} \geq \gamma_i, \forall 1 \leq i \leq U. \end{aligned} \tag{2.44}$$

For simplicity, it is assumed that the set of γ_i in (2.44) is feasible. It can be verified that the SINR constraints in (2.44) are non-convex. In the next section, a technique to reformulate (2.44) in SOCP and SDP forms is presented.

2.9 SOCP and SDP algorithms

In this section, the method developed in [40] to cast (2.44) in a convex form using SOCP and SDP is reviewed. Let

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \vdots \\ \mathbf{h}_U \end{bmatrix} \quad \text{and} \quad \mathbf{W} = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \cdots & \mathbf{w}_U \end{bmatrix}. \tag{2.45}$$

In introducing a real slack variable P_o, (2.44) can be written as [40]:

$$\begin{aligned} \min_{\mathbf{W}, P_o} \quad & P_o \\ \text{subject to} \quad & \frac{|[\mathbf{HW}]_{i,i}|^2}{\sum_{j=1, j\neq i}^{U} |[\mathbf{HW}]_{i,j}|^2 + \sigma^2} \geq \gamma_i, \forall 1 \leq i \leq U \\ & \mathrm{Tr}\left(\mathbf{W}\mathbf{W}^H\right) \leq P_o \end{aligned} \tag{2.46}$$

where $[\mathbf{X}]_{i,j}$ represents the (i,j)-th entry of matrix $\mathbf{X}$. The i-th SINR constraint in (2.46) can be rearrange as:

$$\frac{1}{\gamma_i}\left|[\mathbf{HW}]_{i,i}\right|^2 \geq \sum_{j=1,j\neq i}^{U}\left|[\mathbf{HW}]_{i,j}\right|^2 + \sigma^2. \tag{2.47}$$

Adding $\left|[\mathbf{HW}]_{i,i}\right|^2$ to both sides results in

$$\left(1+\frac{1}{\gamma_i}\right)\left|[\mathbf{HW}]_{i,i}\right|^2 \geq \sum_{j=1}^{U}\left|[\mathbf{HW}]_{i,j}\right|^2 + \sigma^2. \tag{2.48}$$

Equivalently,

$$\left(1+\frac{1}{\gamma_i}\right)\left|[\mathbf{HW}]_{i,i}\right|^2 \geq \left\|\begin{bmatrix}\mathbf{W}^H\mathbf{H}^H\mathbf{e}_i\\ \sigma\end{bmatrix}\right\|^2. \tag{2.49}$$

One can verify the fact that an arbitrary phase rotation can be added to the beamformers without affecting the SINR constraints and objective of (2.46). In other words, if $\mathbf{W}$ is optimal solution to (2.46) then $\mathbf{W}\text{diag}\{e^{j\phi_i}\}$, where ϕ_i for $i=1,2,\cdots,U$ are arbitrary phases, is also an optimal solution. Therefore $\mathbf{W}$ can be selected in such a manner that $[\mathbf{HW}]_{i,i}>0$, i.e., $[\mathbf{HW}]_{i,i}$ can be chosen to be real, for all i without the loss of generality. Since $[\mathbf{HW}]_{i,i}>0,\forall i$, taking the square root of the equation (2.49) leads to

$$\left(1+\frac{1}{\gamma_i}\right)[\mathbf{HW}]_{i,i} \geq \left\|\begin{bmatrix}\mathbf{W}^H\mathbf{H}^H\mathbf{e}_i\\ \sigma\end{bmatrix}\right\|. \tag{2.50}$$

Using vec(.) operator, one can cast the power constraint of (2.46) as

$$p \geq \|\text{vec}(\,\mathbf{W}\,)\| \tag{2.51}$$

where $p=\sqrt{P_o}$. Therefore, problem (2.46) can be reformulated in a SOCP form as

$$\begin{aligned} \min_{\mathbf{W},p} \quad & p \\ \text{subject to} \quad & \left\|\begin{bmatrix}\mathbf{W}^H\mathbf{H}^H\mathbf{e}_i\\ \sigma\end{bmatrix}\right\| \leq \left(1+\frac{1}{\gamma_i}\right)[\mathbf{HW}]_{i,i}, \forall 1\leq i\leq U \\ & \|\text{vec}(\,\mathbf{W}\,)\| \leq p. \end{aligned} \tag{2.52}$$

Using (2.20), the SOCP in (2.52) can be written in a SDP form as

$$\begin{aligned} \min_{\mathbf{W},p} \quad & p \\ \text{subject to} \quad & \begin{bmatrix} \left(1+\frac{1}{\gamma_i}\right)[\mathbf{HW}]_{i,i} & \begin{bmatrix}\mathbf{e}_i^T\mathbf{HW} & \sigma\end{bmatrix} \\ \begin{bmatrix}\mathbf{W}^H\mathbf{H}^H\mathbf{e}_i \\ \sigma\end{bmatrix} & \left(1+\frac{1}{\gamma_i}\right)[\mathbf{HW}]_{i,i}\mathbf{I} \end{bmatrix} \succeq 0, \forall 1 \leq i \leq U \\ & \begin{bmatrix} p & \mathrm{vec}^H(\mathbf{W}) \\ \mathrm{vec}(\mathbf{W}) & p\mathbf{I} \end{bmatrix} \succeq 0. \end{aligned} \tag{2.53}$$

Solving (2.52) or (2.53) provides the optimal beamforming matrix $\mathbf{W}$ and the optimal downlink power as p^2. Beamformer for user i can be obtained as the ith column of $\mathbf{W}$.

2.10 Semidefinite relaxation algorithm

The introduction of the semidefinite relaxation (SDR) technique in early 2000s has provided a capability of obtaining accurate, and sometime near optimal, approximation convex forms from non-convex problems, see [57], [58] and references therein. This section illustrates a method to cast (2.44) in a convex form using the SDR technique.

Let $\mathbf{R}_i = \mathbf{h}_i^H\mathbf{h}_i$ and $\mathbf{F}_i = \mathbf{w}_i\mathbf{w}_i^H$. It is clear that $\mathbf{F}_i$, $\forall 1 \leq i \leq U$, is a positive semidefinite and Hermitian matrix. Further more the rank of the matrix is one. The multiuser downlink beamforming problem in (2.44) can be expressed as

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & \sum_{i=1}^{U} \mathbf{w}_i^H\mathbf{w}_i \\ \text{subject to} \quad & \frac{\mathbf{w}_i^H\mathbf{R}_i\mathbf{w}_i}{\sum_{j=1,j\neq i}^{U} \mathbf{w}_j^H\mathbf{R}_i\mathbf{w}_j + \sigma^2} \geq \gamma_i, \forall 1 \leq i \leq U. \end{aligned} \tag{2.54}$$

Recall the following equality:

$$\mathbf{x}^H\mathbf{A}\mathbf{x} = \mathrm{Tr}\left(\mathbf{A}\mathbf{x}\mathbf{x}^H\right). \tag{2.55}$$

If $\mathbf{A} = \mathbf{I}$ then

$$\mathbf{x}^H\mathbf{x} = \mathrm{Tr}\left(\mathbf{x}\mathbf{x}^H\right). \tag{2.56}$$

Rearrange the ith SINR constraints of (2.54), one can arrive at

$$\left(1+\frac{1}{\gamma_i}\right)\mathrm{Tr}\left(\mathbf{R}_i\mathbf{F}_i\right)-\sum_{j=1,j\neq i}\mathrm{Tr}\left(\mathbf{R}_i\mathbf{F}_j\right)-\sigma^2\geq 0. \tag{2.57}$$

The problem (2.54) can be posed as

$$\begin{aligned}
\min_{\mathbf{F}_i} \quad & \sum_{i=1}^{U}\mathrm{Tr}\left(\mathbf{F}_i\right)\\
\text{subject to} \quad & \frac{1}{\gamma_i}\mathrm{Tr}\left(\mathbf{R}_i\mathbf{F}_i\right)-\sum_{j=1,j\neq i}\mathrm{Tr}\left(\mathbf{R}_i\mathbf{F}_j\right)-\sigma^2\geq 0,\\
& \mathbf{F}_i=\mathbf{F}_i^H\succeq 0,\\
& \mathrm{rank}\left(\mathbf{F}_i\right)=1,\forall 1\leq i\leq U.
\end{aligned} \tag{2.58}$$

The second constraints in (2.58) is to guarantee that $\mathbf{F}_i$, $\forall 1\leq i\leq U$, is a positive semidefinite and Hermitian matrix. Dropping the last constraints in (2.58), i.e., $\mathrm{rank}\left(\mathbf{F}_i\right)=1$, results in a SDP form, i.e.,

$$\begin{aligned}
\min_{\mathbf{F}_i} \quad & \sum_{i=1}^{U}\mathrm{Tr}\left(\mathbf{F}_i\right)\\
\text{subject to} \quad & \frac{1}{\gamma_i}\mathrm{Tr}\left(\mathbf{R}_i\mathbf{F}_i\right)-\sum_{j=1,j\neq i}\mathrm{Tr}\left(\mathbf{R}_i\mathbf{F}_j\right)-\sigma^2\geq 0,\\
& \mathbf{F}_i=\mathbf{F}_i^H\succeq 0,\forall 1\leq i\leq U.
\end{aligned} \tag{2.59}$$

Dropping these rank one constraints not only enlarges the feasible set of the problem (2.58) but also leads to a relaxed SDP problem. This relaxation is referred to as semidefinite relaxation technique. For general nonconvex quadratic problems, solving a SDR problem usually gives an optimal solution with rank of larger than one. In such cases, SDR can only provide a lower bound on the optimal objective function and possibly attain an approximate solution to the original problem [58]. When using SDR results in $\mathbf{F}_i$ solutions with ranks higher than one, a randomization procedure, e.g., see [57], [59] and [60], can be used to find approximate rank-one solutions.

Since (2.44) has a specific structure that it can be turned into a convex form, i.e., as shown in the previous section, strong duality holds for (2.44). Furthermore, it can be shown that the SDR of (2.58), i.e., (2.59), is the Lagrangian dual of the Lagrangian dual of (2.44) [58]. Therefore, (2.59) is exactly equivalent to the original problem (2.44). This fact has been confirmed in [61]. The authors of [61] noticed that the solution to (2.59) always admits rank-one matrices $\mathbf{F}_i, \forall i$, which directly yields the solution to (2.44) using $\mathbf{F}_i = \mathbf{w}_i \mathbf{w}_i^H$.

2.11 Conclusion

In this chapter a general overview of linear programming, convex optimisation, and robust optimisation is summarized. Moreover, the principles of beamforming via linear antenna array along with concepts of second order cone programming and semidefinite programming is reviewed. An optimisation problem to calculate transmit beamformers for multiple active users in a single-cell scenario is sketched. The problem is non-convex due to its non-convex constraints. Two methods are presented to transform the problem into second order cone programming and semidefinite programming forms, which can be effectively solved by available optimisation packets.

Chapter 3

Secondary Spectrum Access and Cell-Edge Coverage in Cognitive Cellular Networks

This chapter focuses on the problem of cell-edge user coverage in the context of cognitive radio networks operating within the vicinity of primary cell borders. Two strategies are introduced such that the primary cell-edge users get assisted by the cognitive BS to receive a consistent QoS due to their long distance from the primary BS. In return, the cognitive BS is rewarded by using the same spectrum that has already been allocated to the primary user's link to serve a group of cognitive users. In the first strategy called as cooperative, the cognitive BS relays the primary cell-edge user's data, sent by the primary BS, through spatial multiplexing and beamforming, while transmitting towards its cognitive users. In the second strategy known as soft interference shaping, the cognitive BS serves cognitive users as well as primary cell-edge users by spatial multiplexing and beamforming, while forming controlled nulls towards the primary users located outside but within the close vicinity of the cognitive cell border. This technique is done to avoid the interference towards the primary users surrounding the cognitive cell border.

3.1 Introduction

Cognitive radio has been proposed as a promising technology for improving the utilisation efficiency of the radio spectrum [2]. In a cognitive system, the cognitive (secondary) user seeks to overcome the spectral shortage problem by using the primary user's bandwidth without causing any interference [1], [62]. In a spectrum sharing scenario, the secondary user can co-exist with the primary user all the time as long as the interference power received by the primary user is less than a threshold, which is determined by the QoS of the Primary user [63], [64], [65].

In order to fully utilise the limited spectrum, the spectrum sharing strategy between the primary and secondary users is an important issue. A common assumption in cognitive radio systems is that the licensed users which own the spectrum rights are unaware of the presence of secondary users. Hence the burden of interference management relies mainly on the secondary system. In particular, either there is a maximum interference level that the primary system is willing to tolerate, and the secondary powers/activity are to be adjusted within this constraint, hence both primary and secondary users transmit in the same band, or secondary users are allowed to opportunistically access the spectrum on the basis of no-interference to the primary (licensed) users. These two paradigms fit into what is commonly known as hierarchical-access schemes, referring to the fact that secondary users need to fulfill the constraints imposed by the primary user. Two approaches to spectrum sharing have been addressed: spectrum overlay and spectrum underlay.

In spectrum overlay paradigm, a secondary spectrum user is constrained to avoid interfering with an active primary spectrum user via spectrum sensing, adaptive detection and allocation of unused spectrum portions by the primary users. Then, the detected spectrum opportunities can be shared within a network of cognitive users [66], [67].

In spectrum underlay paradigm, a secondary spectrum user is restricted in terms of transmitting power level so that its inflicted interference on a receiving point in a primary network falls below the noise floor [68].

Power allocation and beamforming problems for multi-user systems have been widely studied over the past decade, e.g., [69] and [61]. Beamforming is a space-division-multiple-access technique where multiple antennas and advanced spatial signal processing are used to serve multiple co-channel users. In linear beamforming, for example, the data stream for each user is modulated by a precoding vector, i.e., a spatial signature, before going through these transmit antennas. By careful selection of precoding vectors, mutual interference amongst different streams can be mitigated or even removed [70]. Therefore, using beamforming yields improvements in transmission range, rate and power efficiency [71]. However, as the user moves towards the cell-edge areas, dominated by severe intercell interference, the technique cannot assure and maintain a consistent level of data rate to the user. Recently, the idea of multi-cell processing has promised a solution to the cell-edge problem by allowing intercell cooperation, e.g., [72–78]. The coordinated design of precoding vectors for multiple coordinated cells results in significant improvements of throughput with respect to uncoordinated design, e.g., [74], [70]. Full cooperation amongst BSs within a cluster offers significant sum throughput and cell-edge user rate gains [79], [80]. Under the context of a distributed antenna system, the authors of [81] show that jointly designing the transmit preprocessing matrix of all the cooperative remote antennas combined with fractional frequency reuse is capable of achieving an increased throughput for the entire cell-edge area. Based on several early papers on optimal downlink beamforming and power control [69], [82], there has been a more recent work on signal-to-interference-and-noise ratio (SINR) balancing for designing downlink beamformers [83]. In [84] optimal downlink beamforming in a communication system has been formulated to minimise the transmission power subject to QoS constraints, using rank constrained solutions.

The transmit beamformer design for cognitive radio networks at the secondary BS has been used to control the interference level, due to the secondary transmission, at the primary users. By doing so, the Signal-to-Noise Ratio (SNR) of the secondary user can be improved, yet the interference power to the primary user is limited to a certain acceptable level. In [65], spatial diversity has been exploited in downlink to improve the channel capacity between secondary users, while imposing constraints on the secondary user transmit power and the primary user interference power. In [85] the author has considered the co-existence between cognitive users and a primary user, where an efficient transmit beamforming technique combined with user selection is proposed to maximize the downlink throughput and satisfy the SINR constraint as well as limit interference to the primary user. In [86] an underlay hierarchical cognitive radio environment is considered, where two cases were proposed respective to the interference tolerance by the primary users. In the first case, it was assumed that a limited amount of interference is tolerated by the primary systems originating from the secondary systems. This constraint was modeled such that each secondary user is penalized by a quantity that is proportional to the generated interference rate from that user on the primary users. In the second case, interference from secondary systems is not tolerated by the primary systems. For both cases optimal beamforming vectors were characterised which satisfied the constraints. In another scenario considered in [87], two secondary users exchange their information through a cognitive relay while beamforming and power allocation are employed to enhance the achievable sum rate. Cooperative communication based on relaying between secondary users in the presence of one or more primary links is considered in [88].

This chapter focuses on downlink cognitive communication within a cellular network. A cluster of primary cells sharing the same bandwidth is considered. In addition, small cognitive cells are located within the vicinity of the primary cell borders. First, a cooperative strategy is introduced where the secondary BS relays data, received from the primary BS, to a primary cell-edge user, while transmitting data to the secondary users within the cognitive cell, through spatial multiplexing. Second, a soft interference shaping strategy is proposed where the secondary BS can focus its radiation pattern along the direction of the users within the cognitive cell, while forming nulls with controlled depths towards the primary users located outside but within the close vicinity of the cognitive cell border. In other words, the primary users surrounding the cognitive cell border do not tolerate interference level above an allowed threshold from the secondary system. The optimisation problem is formulated in the standard Semidefinite programming (SDP) which is a subfield of a Matlab-based modeling system for convex optimisation (CVX) [6], [89].

This chapter is organized as follows. Section 3.2 introduces the system model. Section 3.3 describes the cognitive strategies. Numerical results for simulated scenarios are presented in section 3.4. Section 3.5, draws some concluding remarks.

3.2 System model

Fig. 3.1 shows a multi-cell multi-user spatial multiplex system with two large primary cells and a small cognitive cell located within the vicinity of the primary cells border. K_p single antenna primary users, K_s single antenna secondary users within the cognitive cell, and a linear antenna array of N elements per sector at the secondary BS is considered. The assumption is used that each user is surrounded by Q random local-scatterers within the radius of 100 wavelengths [90] and there is no Line-of-sight (LoS) transmission from the secondary BS to all users. By demonstrating our algorithm without the LOS transmission, the algorithm's performance will be shown in the worst case. Each user receives signals from Q local scatterers excited by the secondary BS. It is considered that each user and its local scatterers are at far-field distances from the secondary BS. Thus, wavefronts from the serving-array antenna of the secondary

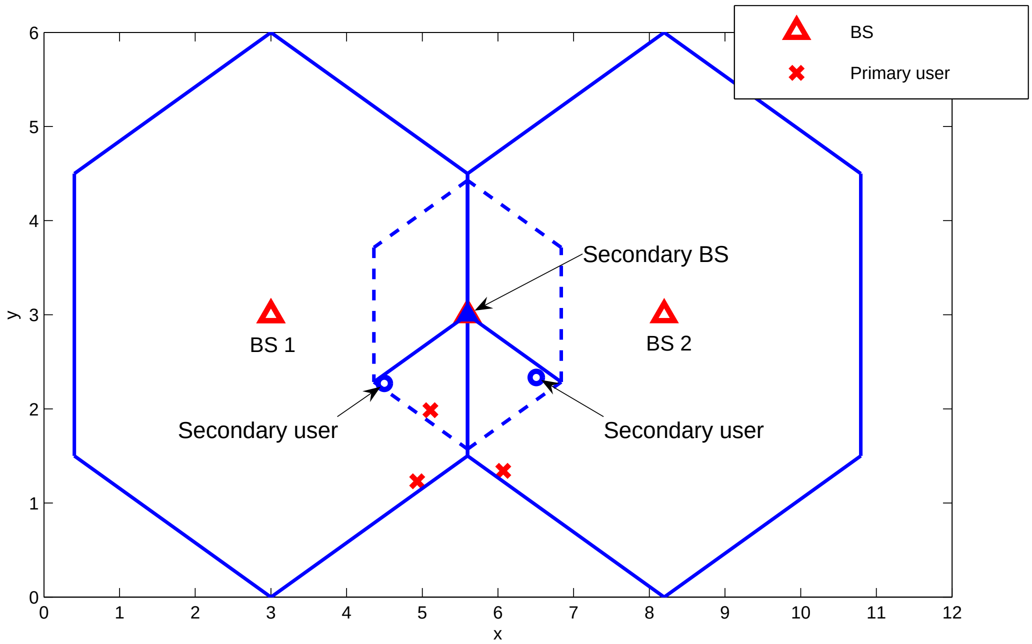

Figure 3.1: Cognitive cellular network with two primary cells and a cognitive cell located on the primary cells border.

BS will hit all Q scatterers. The spacing between 2 elements of an array is negligible in comparison with the distance of the secondary BS from the scatterers. Therefore, rays departing from elements of one array to one direction, i.e., one scatterer, have the same fading gain. Moreover, the maximum time of the wavefront traveling along one array is assumed to be much smaller than the reciprocal of the bandwidth of all transmitted signals.

The interference caused by the primacy BSs on the secondary users is ignored, due to the fact that the primary BSs are farther than the secondary BS to their secondary users. However, one may consider that the secondary BS can pre-subtract the interference caused by the primary BSs in the secondary users by Dirty Paper Coding (DPC) [91]. Applying DPC becomes a more realistic assumption, in particular, when the secondary and primary BSs are located in close vicinity such that the primary BS information can be correctly revealed to the secondary BS via a reliable backhual link. The details of well researched area of DPC can be found in literature, e.g. [92] and

[93], and its treatment is beyond the scope of this chapter. In the absence of a reliable backhaul link, any residual interference caused by the primary BS can be treated as additional noise on the secondary users.

Let

$$\mathbf{x_m} = [x_m(1)x_m(2)\dots x_m(N)]^T, \tag{3.1}$$

where $x_m(k)$ is the transmitted signal by the m^{th} antenna element of the secondary BS, and s_{i_s}, $i_s = 1, \dots, K_s$, to be the intended symbol for user i_s. The transmitted signal by the secondary BS can be written as

$$\mathbf{x} = \sum_{i=1}^{K} \mathbf{w}_{i_s} s_{i_s}, \tag{3.2}$$

$\mathbf{w}_{i_s}$ is $3N \times 1$ transmit beamforming vector for user i_s at the secondary BS, s_{i_s} denotes the information signal for user i_s. Hence, one can write the received signal by the secondary user i_s, i.e., $i_s = 1, \dots, K_s$, within the cognitive cell as

$$\mathrm{y}_{i_s} = \sum_{n=1}^{Q} \mathbf{h}_{i_s n} \mathbf{w}_{i_s} \mathrm{s}_{i_s} + \sum_{n=1}^{Q} \sum_{j=1, j\neq i_s}^{K} \mathbf{h}_{jn} \mathbf{w}_j \mathrm{s}_j + \mathrm{n}_{i_s}, \tag{3.3}$$

where $n \in \{1, 2, 3, \cdots Q\}$ is the number of random local scatters surrounding secondary user i_s, $\mathbf{h}_{i_s n} = [\mathrm{h}_{i_s n}(1) \ \ \mathrm{h}_{i_s n}(2) \cdots \mathrm{h}_{i_s n}(N)]$ is the vector channel with $\mathrm{h}_{i_s n}(m)$ being the channel between the m^{th} element, i.e., $m \in \{1, 2 \cdots N\}$, of the secondary BS and scatterer n of user i_s, K is the total number of the secondary and primary cell-edge users located inside the cognitive cell, and n_{i_s} indicates any residual intercell interference and white Gaussian noise at user i_s. It is assumed that n_{i_s} is zero mean complex Gaussian with variance $\sigma_{i_s}^2$, i.e., $\mathrm{n}_{i_s} \sim \mathcal{CN}(0, \sigma_{i_s}^2)$.

It is considered that the cognitive cell is divided into 3 sectors with a linear antenna array of N elements per each sector. The secondary BS allocates all the users within the cognitive cell to the nearest array. It can be shown that

$$\mathbf{h}_{i_s n} = \left[a_{i_s n}^1 \mathbf{h}_{i_s n}^1 \ a_{i_s n}^2 \mathbf{h}_{i_s n}^2 \ a_{i_s n}^3 \mathbf{h}_{i_s n}^3\right], \tag{3.4}$$

where $a^q_{i_s n}$ is either 1 if user i_s is allocated, or 0 if user i_s is not allocated to be served by sector $q \in \{1, 2, 3\}$ of the cognitive cell. Furthermore, it can be stated that

$$\mathbf{h}^q_{i_s n} = \left[h^q_{i_s n,(q-1)N+1} \quad h^q_{i_s n,(q-1)N+2} \cdots h^q_{i_s n,(q-1)N+N} \right], \tag{3.5}$$

where $h^q_{i_s n,(q-1)N+m}$ is the channel between the m^{th} element, i.e., $m \in \{1, 2, 3, \cdots N\}$, of the array in sector q and scatterer n of user i_s.

The channel model can be stated as

$$h^q_{i_s n,(q-1)N+m} = c l^{-\frac{\alpha}{2}}_{i_s n} \left[g_n e^{j\beta_n} e^{j\frac{2\pi}{\lambda}\left[d(m-1)\sin(\theta^q_{i_s}+\phi_n)\right]} \right], \tag{3.6}$$

where $c l^{-\frac{\alpha}{2}}_{i_s n}$ is the path loss coefficient between the secondary BS and scatterer n of user i_s; α is the path loss exponent; $l_{i_s n}$ is the distance between secondary BS and scatterer n of secondary user i_s; c is a constant factor, g_n is the Rayleigh fading gain, β_n is the phase delay at scatterer n, d is the spacing between array antenna elements within a sector, $\theta^q_{i_s}$ is the angle of departure at the q^{th} sector of the secondary BS for user i_s, and, ϕ_n is the angular spread of scatterer n with respect to $\theta^q_{i_s}$. It is assumed that the local scatters are distributed randomly around each user and the resulting angle spread has a normal distribution σ, i.e., $\phi_n \sim \mathcal{N}(0, \sigma^2)$.

Similarly the channel between the m^{th} element of the q^{th} sector of secondary BS and scatterer n of primary cell-edge user i_p is given as

$$h^q_{i_p n,(q-1)N+m} = c l^{-\frac{\alpha}{2}}_{i_p n} \left[g_n e^{j\beta_n} e^{j\frac{2\pi}{\lambda}\left[d(m-1)\sin(\theta^q_{i_p}+\phi_n)\right]} \right]. \tag{3.7}$$

The channel covariance matrix between secondary BS and secondary user i_s is defined as

$$\mathbf{R}_{i_s} = \mathbf{E}\left(\sum_{n=1}^{Q} \sum_{t=1}^{Q} \mathbf{h}^{\mathrm{H}}_{i_s n} \mathbf{h}_{i_s t} \right). \tag{3.8}$$

It has been shown (See Appendix A for proof) that the $(k, m)^{th}$ element of the channel covariance matrix between secondary BS and secondary user i_s is calculated as

$$\mathbf{R}_{i_s}\left[k,m\right]=\sum_{n=1}^{Q} c l_{i_s n}^{-\frac{\alpha}{2}} \delta_g e^{\frac{j2\pi d}{\lambda}\left[(m-k)\sin\theta_{i_s}^q\right]} \times e^{-2\left[\frac{\pi d\sigma}{\lambda}(m-k)\cos\theta_{i_s}^q\right]^2}, \tag{3.9}$$

where $k, m \in \{1, 2 \cdots 3N\}$, $\mathbf{E}\left[|g_n|^2\right] = \delta_g$, and $|g_n|^2$ has Chi-square distribution with two degrees of freedom. Similarly the channel covariance matrix between secondary BS and a primary user can be derived from (3.7).

Assuming that the average energy of symbol constellation is normalised to unity, i.e., $\mathbf{E}_{s_{i_s}}\left[|s_{i_s}|^2\right] = 1$, the SINR at the secondary user i_s can be expressed as

$$\mathrm{SINR}_{i_s} = \frac{\mathbf{w}_{i_s}^H \mathbf{R}_{i_s} \mathbf{w}_{i_s}}{\sum_{j=1, j\neq i_s}^{K} \mathbf{w}_j^H \mathbf{R}_{i_s} \mathbf{w}_j + \sigma_{i_s}^2},\ 1 \leq i_s \leq K_s. \tag{3.10}$$

Note that since the cognitive cell is located within the vicinity of the primary cells border, the intercell interference is regarded as a part of the background noise, i.e, $\sigma_{i_s}^2$ in (3.10).

3.3 Problem formulation

3.3.1 Cooperative strategy

The distance between primary cell-edge users, located inside the cognitive cell, and primary BS is large. Therefore, the primary BS cannot fully support the primary cell-edge user i_p. On the other hand, the interference towards the unintended secondary users caused by the primary BS is considerably weak compared to the signal power sent by the secondary BS. Therefore, the secondary BS communicates with the secondary users, using the licensed primary spectrum. In return, the cognitive BS relays, i.e., Decode-and-Forward (DF) relay type, the primary cell-edge users' data, sent by the primary BS, through spatial multiplexing, while transmitting towards the secondary users. By using this approach, both systems can benefit times of need.

The signal received by primary cell-edge user i_p, i.e., $i_p \in \{1, 2 \cdots K_p\}$, within the cognitive cell is written as

$$\mathrm{y}_{i_p} = \sum_{n=1}^{Q} \mathbf{h}_{i_p n} \mathbf{w}_{i_p} \mathrm{s}_{i_p} + \sum_{n=1}^{Q} \sum_{j=1, j \neq i_p}^{K} \mathbf{h}_{jn} \mathbf{w}_j \mathrm{s}_j + \mathrm{n}_{i_p}, \tag{3.11}$$

where $\mathbf{h}_{i_p n} \mathbf{w}_{i_p} \mathrm{s}_{i_p}$ is the information received at user i_p sent by the secondary BS, $\sum_{j=1, j \neq i_p}^{K} \mathbf{h}_{jn} \mathbf{w}_j \mathrm{s}_j$ is the interference generated from all users within the cognitive cell, and n_{i_p} indicates any residual intercell interference and white Gaussian noise at user i_p. It is assumed that n_{i_p} is zero mean complex Gaussian with variance $\sigma_{i_p}^2$, i.e., $\mathcal{CN}(0, \sigma_{i_p}^2)$.

Furthermore, the SINR of the primary cell-edge user i_p within the cognitive cell with can be written as

$$\mathrm{SINR}_{i_p} = \frac{\mathbf{w}_{i_p}^H \mathbf{R}_{i_p} \mathbf{w}_{i_p}}{\sum_{j=1, j \neq i_p}^{K} \mathbf{w}_j^H \mathbf{R}_{i_p} \mathbf{w}_j + \sigma_{i_p}^2}, \; 1 \leq i_p \leq K_p, \tag{3.12}$$

where $\mathbf{R}_{i_p}$ is the channel covariance matrix between secondary BS and primary cell-edge user i_p within the cognitive cell.

The beamformer design problem in this scenario consists of minimising the total power at the secondary BS subject to SINR constraints at secondary and primary cell-edge users. This means that secondary BS minimises the total power, by using the same spectrum that has already been allocated to the primary user's link. In exchange, the secondary BS spatially multiplex the information sent by the primary BS to the primary cell-edge user. Using (3.10) and (3.12), The problem is formulated as

$$\begin{aligned} \min_{\mathbf{w}_{i_s}} \quad & \sum_{i_s=1}^{K} \mathbf{w}_{i_s}^H \mathbf{w}_{i_s} \\ \text{subject to} \quad & \frac{\mathbf{w}_{i_s}^H \mathbf{R}_{i_s} \mathbf{w}_{i_s}}{\sum_{j=1, j \neq i_s}^{K} \mathbf{w}_j^H \mathbf{R}_{i_s} \mathbf{w}_j + \sigma_{i_s}^2} \geq \gamma_s, \; 1 \leq i_s \leq K_s, \\ & \frac{\mathbf{w}_{i_p}^H \mathbf{R}_{i_p} \mathbf{w}_{i_p}}{\sum_{j=1, j \neq i_p}^{K} \mathbf{w}_j^H \mathbf{R}_{i_p} \mathbf{w}_j + \sigma_{i_p}^2} \geq \gamma_p, \; 1 \leq i_p \leq K_p, \end{aligned} \tag{3.13}$$

where γ_s and γ_p are the required SINR at secondary user i_s and primary cell-edge user i_p respectively.

The constraints involve quadratic non-convex functions of variables. However, this can be modified into the SDP standard formulation. This can be done by changing the vector variables $\mathbf{w}_{i_s}$ into matrix variables $\mathbf{F}_{i_s}$.

Consider the following definitions

$$\begin{aligned} \mathbf{F}_{i_s} &= \mathbf{w}_{i_s}\mathbf{w}_{i_s}^H, \\ \mathbf{F}_{i_p} &= \mathbf{w}_{i_p}\mathbf{w}_{i_p}^H. \end{aligned} \tag{3.14}$$

Using the following conditions

$$\begin{aligned} \mathbf{w}_{i_s}^H\mathbf{w}_{i_s} &= \mathrm{Tr}\left[\mathbf{w}_{i_s}\mathbf{w}_{i_s}^H\right], \\ \mathbf{w}_{i_p}^H\mathbf{w}_{i_p} &= \mathrm{Tr}\left[\mathbf{w}_{i_p}\mathbf{w}_{i_p}^H\right], \\ \mathbf{w}_{i_s}^H\mathbf{R}_{i_s}\mathbf{w}_{i_s} = \mathrm{Tr}\left[\mathbf{R}_{i_s}\mathbf{w}_{i_s}\mathbf{w}_{i_s}^H\right] &= \mathrm{Tr}\left[\mathbf{R}_{i_s}\mathbf{F}_{i_s}\right], \\ \mathbf{w}_{i_p}^H\mathbf{R}_{i_p}\mathbf{w}_{i_p} &= \mathrm{Tr}\left[\mathbf{R}_{i_p}\mathbf{F}_{i_p}\right], \end{aligned} \tag{3.15}$$

and introducing a slack variable P_o, problem (3.13) can be reformulated as

$$\begin{aligned} \min_{\mathbf{F}_{i_s},\mathrm{P}_o} \quad & \mathrm{P}_o \\ \text{subject to} \quad & \left(1+\frac{1}{\gamma_s}\right)\mathrm{Tr}\left[\mathbf{R}_{i_s}\mathbf{F}_{i_s}\right] - \sum_{j=1}^{K}\mathrm{Tr}\left[\mathbf{R}_{i_s}\mathbf{F}_j\right] - \sigma_{i_s}^2 \geq 0, \\ & \left(1+\frac{1}{\gamma_p}\right)\mathrm{Tr}\left[\mathbf{R}_{i_p}\mathbf{F}_{i_p}\right] + \sum_{j=1}^{K}\mathrm{Tr}\left[\mathbf{R}_{i_p}\mathbf{F}_j\right] - \sigma_{i_p}^2 \geq 0, \\ & \mathrm{P}_o - \sum_{i_s=1}^{K}\mathrm{Tr}\left[\mathbf{F}_{i_s}\right] \geq 0, \\ & \mathbf{F}_{i_s} = \mathbf{F}_{i_s}^H \succeq 0. \end{aligned} \tag{3.16}$$

The fourth constraint in (3.16) is to satisfy the condition that $\mathbf{F}_{i_s}$ is Hermitian positive semidefinite. The problem stated in (3.16) is in standard SDP form and equivalent to the one in (3.13) if $\mathrm{rank}[\mathbf{F}_{i_s}] = 1$. However, it was shown in [94] that if (3.16) is feasible, there is at least one solution satisfying the condition of $\mathrm{rank}[\mathbf{F}_{i_s}] = 1$ for all $i_s \in \{1, 2 \cdots K\}$. Let ϵ_{i_s} and X_{i_s} be eigenvalue and corresponding eigenvector of the rank 1 matrix $\mathbf{F}_{i_s}$ respectively. Then, it can be easily shown that the beamforming vector for user i_s is given as

$$\mathbf{w}_{i_s} = \sqrt{\epsilon_{i_s}} X_{i_s}. \tag{3.17}$$

One can find the square root of $\mathbf{F}_{i_s}$ as follows

$$\begin{aligned}\mathbf{F}_{i_s} &= X_{i_s} \epsilon_{i_s} \epsilon_{i_s}^H \\ &= (\sqrt{X_{i_s}} \epsilon_{i_s})(\sqrt{X_{i_s}} \epsilon_{i_s})^H \\ &= \mathbf{w}_{i_s} \mathbf{w}_{i_s}^H\end{aligned} \tag{3.18}$$

An alternative scenario can be shown in Fig. 3.2, where a large primary hexagonal cell includes a small cognitive cell. The cognitive cell is located at the edge of the primary cell which contains a secondary BS and K_s secondary users. The primary users within the cognitive cell can not be supported fully by primary BS, due to the long distance. Using the same problem formulation in (3.13), it can be shown that the cognitive system has the permission to operate within the primary cell using the primary user's bandwidth. In return, the cognitive cell supports the primary cell-edge users by spatially multiplexing the primary user's information sent by primary BS.

3.3.2 Soft interference shaping strategy

In this section, it is assumed that the primary users located outside but within the close vicinity of the cognitive cell border do not tolerate interference level above an allowed threshold from the secondary system. The secondary BS designs beamforming vectors to minimise its transmit power supporting all users within the cognitive cell, i.e, secondary and primary cell-edge users, while forming nulls with controlled depths towards the primary users, supported by the primary BS. Given the channels to primary users outside but within the close vicinity of the cognitive cell borders, the secondary BS is to form nulls in the direction of these channels, i.e.,

$$\sum_{i_s=1}^{K} \mathbf{w}_{i_s}^H \mathbf{R}_{i_p} \mathbf{w}_{i_s} \leq I, \ 1 \leq i_p \leq K_p, \tag{3.19}$$

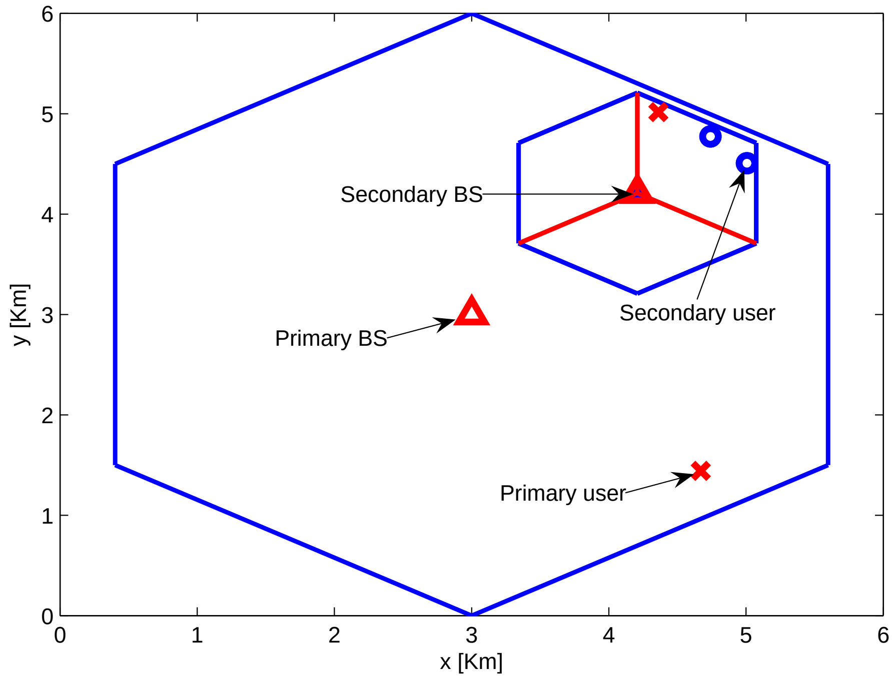

Figure 3.2: An alternative cognitive cellular network with a cognitive cell located within the primary cell and close to the cell-edge.

where $\mathbf{R}_{i_p}$ is the channel covariance matrix between secondary BS and primary users.

The Channel-State-Information (CSI) required by the secondary BS is the channel to the primary user. Assuming channel reciprocity, the information can be locally obtained by the secondary BS during the uplink phase of the primary user. In other words, when the primary user is communicating uplink with the primary BS, the secondary BS can estimate its channel to the primary user and use it for soft interference shaping. In the absence of channel reciprocity, i.e., Frequency-Division Duplexing (FDD) transmission, the CSI should be provided via a feedback link between the secondary BS and the primary user.

The beamformer design problem in this scenario consists of minimising the total power at secondary BS subject to a SINR constraint on all users within the cognitive cell and a soft interference shaping constraint on the primary users located outside but within the close vicinity of the cognitive cell. The problem is formulated as follow

$$\begin{aligned}
\min_{\mathbf{w}_{i_s}} \quad & \sum_{i_s=1}^{K} \mathbf{w}_{i_s}^H \mathbf{w}_{i_s} \\
\text{subject to} \quad & \frac{\mathbf{w}_{i_s}^H \mathbf{R}_{i_s} \mathbf{w}_{i_s}}{\sum_{j=1, j\neq i_s}^{K} \mathbf{w}_j^H \mathbf{R}_{i_s} \mathbf{w}_j + \sigma_{i_s}^2} \geq \gamma_s, \; 1 \leq i_s \leq K_s, \\
& \sum_{i_s=1}^{K} \mathbf{w}_{i_s}^H \mathbf{R}_{i_p} \mathbf{w}_{i_s} \leq I, \; 1 \leq i_p \leq K_p,
\end{aligned} \tag{3.20}$$

where I is the interference threshold for the primary user. Following the same method in the previous section with a slack variable P_o, provides

$$\begin{aligned}
\min_{\mathbf{F}_{i_s}, \mathrm{P}_o} \quad & \mathrm{P}_o \\
\text{subject to} \quad & \left(1 + \frac{1}{\gamma_s}\right) \mathrm{Tr}\left[\mathbf{R}_{i_s} \mathbf{F}_{i_s}\right] - \sum_{j=1}^{K} \mathrm{Tr}\left[\mathbf{R}_{i_s} \mathbf{F}_j\right] - \sigma_{i_s}^2 \succeq 0, \\
& I - \sum_{i_s=1}^{K} \mathrm{Tr}\left[\mathbf{R}_{i_p} \mathbf{F}_{i_s}\right] \geq 0; \\
& \mathrm{P}_o - \sum_{i_s=1}^{K} \mathrm{Tr}\left[\mathbf{F}_{i_s}\right] \succeq 0, \\
& \mathbf{F}_{i_s} = \mathbf{F}_{i_s}^H \succeq 0.
\end{aligned} \tag{3.21}$$

In addition, to ensure that the interference at each of the secondary users is directly removed from the signal without any further processing, one may enforce orthogonality of the transmitted signals at the BS by setting $\mathrm{Tr}\left[\mathbf{R}_{i_s} \mathbf{F}_j\right] = 0$ in (3.21). Finally, an equivalent form of the original problem (3.20) in the standard SDP form as follows

$$
\begin{aligned}
\min_{\mathbf{F}_{i_s},\mathrm{P}_o} \quad & \mathrm{P}_o \\
\text{subject to} \quad & \left(1+\frac{1}{\gamma_s}\right)\mathrm{Tr}\left[\mathbf{R}_{i_s}\mathbf{F}_{i_s}\right]-\sigma_{i_s}^2 \geq 0, \\
& \sum_{j=1}^{K}\mathrm{Tr}\left[\mathbf{R}_{i_s}\mathbf{F}_j\right]=0, \\
& I-\sum_{i_s=1}^{K}\mathrm{Tr}\left[\mathbf{R}_{i_p}\mathbf{F}_{i_s}\right]\geq 0, \\
& \mathrm{P}_o-\sum_{i_s=1}^{K}\mathrm{Tr}\left[\mathbf{F}_{i_s}\right]\succeq 0, \\
& \mathbf{F}_{i_s}=\mathbf{F}_{i_s}^H \geq 0.
\end{aligned}
\tag{3.22}
$$

3.4 Simulation results

This section presents some numerical examples illustrating the performances of our proposed schemes and finally compared together. For simplicity, the scenario is assumed with a single secondary BS serving two secondary users and a single primary cell-edge user within the cognitive cell. It is also assumed that there is one primary user per primary cell which is located in the outer part of the cognitive cell, but within the close vicinity. Note that each user is equipped with a single antenna.

As shown in Fig. 3.1, secondary and primary cell-edge users within the cognitive cell are located in sector 3, i.e. q=3. The experiment is done with a single scatterer, i.e, $Q = 1$. The angular spread of local scatters surrounding the users is to be assumed 2 degrees. The spacing distance between the array elements is $\lambda/2$. The carrier frequency is 2 GHz. The noise variance plus the intercell interference is set to 1. In this simulation, SeDuMi solver under optimisation solver CVX [6], [89] is used to attain the optimal solution for the problems stated in (3.16) and (3.21).

The azimuth directions (angle of propagation with respect to the antenna array broadside) of the users as well as the angular spread due to the local scatters corresponding to the sector of the secondary BS can be estimated using the algorithm described in [95]. Furthermore, the resolution of the estimates depends on the number of antenna elements in each sector of the secondary BS. The figures in [95], show the accuracy of these estimations when the number of antenna elements varies between 4 to 12. In the simulations is has been considered that the secondary BS has 8 antenna array elements per sector. Tables 3.1 and 3.2 show the Angle-of-Departure (AoD) and the normalised distance between the secondary BS and secondary/primary cell-edge users respectively for both cases stated in (3.16) and (3.21).

Since the users are located in sector 3, the optimal radiation pattern of array 3 of the secondary BS is shown in Fig. 3.3. In this figure, the total and individual users' beam patterns in the case that optimal beamforming in (3.16) is cooperative strategy are shown. The target SINR for primary cell-edge and secondary users are arbitrary given values that are required by the user. Therefore, in this simulation, we have set $\gamma_p = 10dB$, $\gamma_s = 15dB$. As is clearly evident from Fig.(3.3), users are supported only by nearby arrays (in this case array 3). This is due to the total power minimising objective function and the fact that the simulation takes into a account the path loss. The interference generated at the location of the two external primary users outside of the border of the cognitive cell ($\theta = 20.60^o$ and $\theta = -15.84^o$) is high. The details of this distribution is shown in table 3.1.

Fig. 3.4 shows the optimal radiation pattern of the secondary BS when soft interference shaping constraint is included to protect the primary users surrounding the border of the cognitive cell from interference, stated in (3.21). In this part, the primary cell-edge user within the cognitive cell are supported by the secondary BS, through spatial multiplexing. In this case, as expected, the secondary BS form a null in the direction of each primary user's channel surrounding the border of the cognitive cell. This can be seen in the beam pattern towards the direction of the two external primary users, i.e., $\theta = 20.60^o$ and $\theta = -15.84^o$ is low. The details of this distribution are shown in table 3.2 and 3.3.

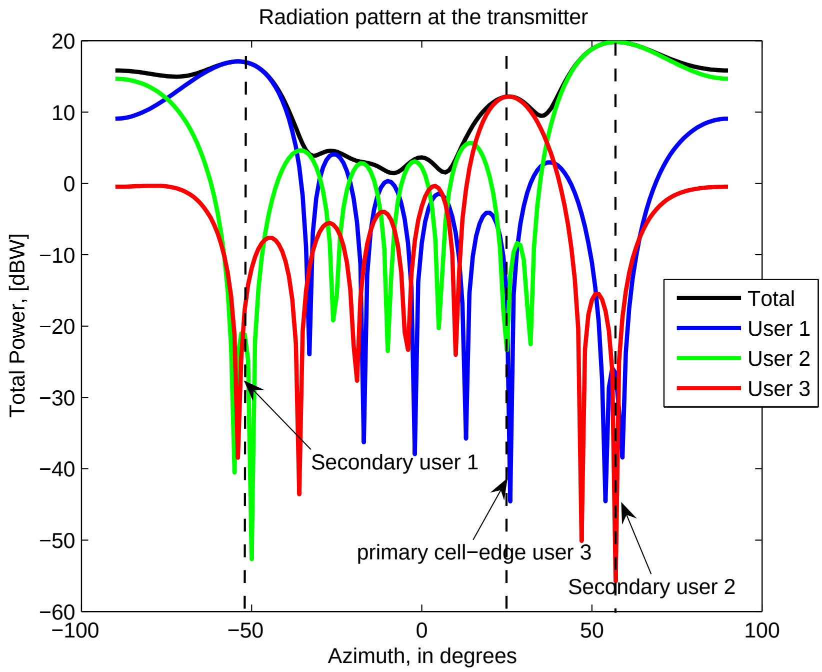

Figure 3.3: Radiation pattern at array 3 of secondary BS with cooperative strategy, $\gamma_s = 15dB$, and $\gamma_p = 10dB$.

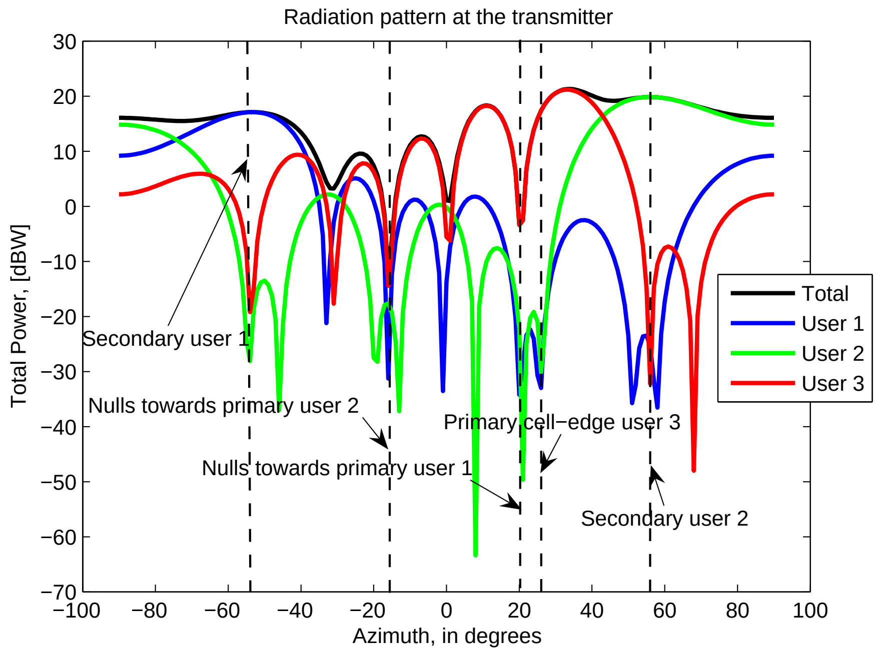

Figure 3.4: Radiation pattern at array 3 of secondary BS with soft interference shaping strategy, $\gamma_s = 15dB$, and $I = -30dBW$.

	AoD (degree)	Distance
Secondary User 1	-53.73	1.12
Secondary User 2	56.46	1.32
Primary cell-edge User	25.78	1.12

Table 3.1: AoD and the distance (normalised to the radius of the cognitive cell) between each user and the secondary BS, using cooperative scheme.

	AoD (degree)	Distance
Secondary User 1	-53.73	1.12
Secondary User 2	56.46	1.32
Primary cell-edge User	25.78	1.12
Primary User 1	20.60	1.89
Primary User 2	-15.844	1.72

Table 3.2: AoD and the distance (normalised to the radius of the cognitive cell) between each user and the secondary BS, using soft interference shaping scheme.

	Primary user 1 location	Primary user 2 location
PG secondary user 1	18.54	30.11
PG secondary user 2	15.21	21.41
PG primary cell-edge user	6.011	6.99

Table 3.3: Power gain (PG) in dBW for each user within the cognitive cell at the location of the two primary outer-cell users, i.e., AoD = 20.06° and -15.844°, respectively.

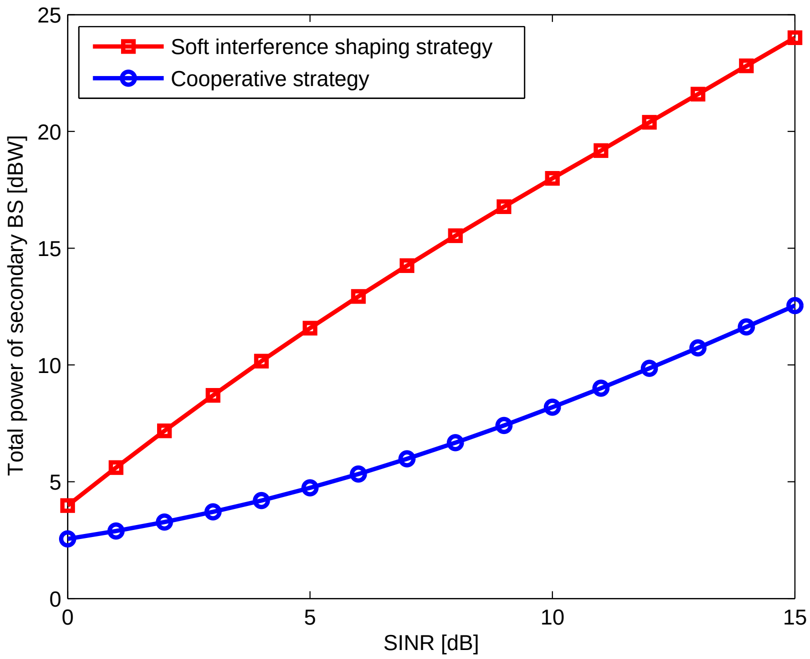

Figure 3.5: Minimal transmission power versus the secondary SINR, $\gamma_s = 15dB$, and $I = -30dBW$.

Fig. 3.5 displays the total power versus the secondary target SINR for both cases, i.e. cooperative strategy and soft interference shaping strategy. Monte-Carlo simulations, where channel generation are 1000 times. It can be observed that the required power is larger with soft interference shaping constraint. The total transmit power of the secondary BS of the cooperative scheme increases from 2.55 to 12.54 dBW when the target SINR varies from 0 to 15 dB while with the same range of SINR, the soft interference shaping scheme requires the total transmit power from 3.97 to 24.01 dBW. When the case is cooperative strategy, the minimal transmit power is lower but with no control on the radiation power towards the primary users; in the case the soft interference shaping strategy, the minimal transmit power is higher but being able to keep the radiation power towards the primary users under the given interference threshold.

Fig.(3.6) shows the performance of soft interference shaping strategy, i.e., 3.22, in minimising the total transmit power at the secondary BS as a function of the angular

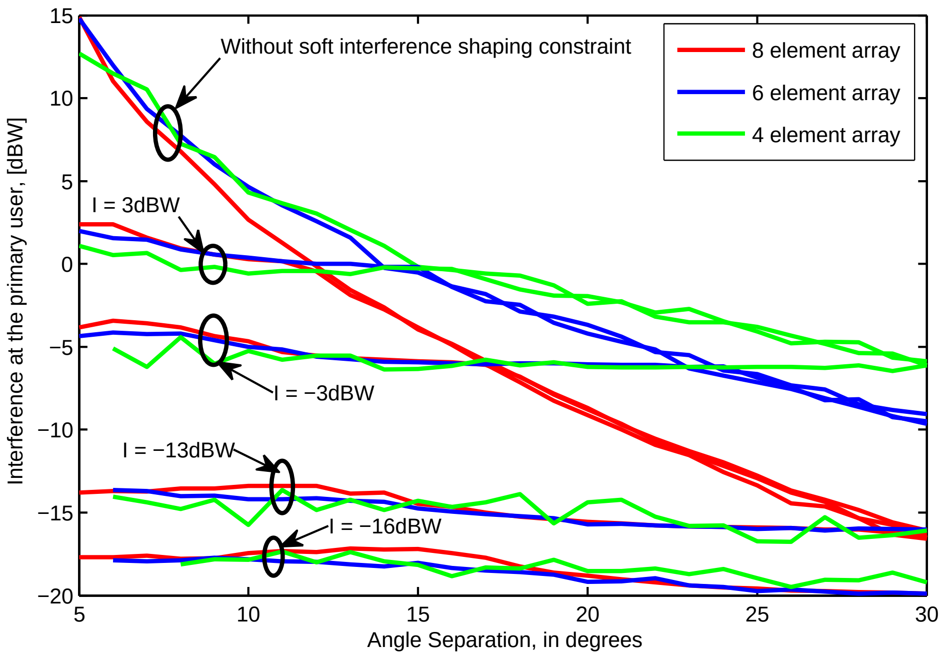

Figure 3.6: Minimised transmit power at the secondary BS versus angular separation between the secondary users as a function of allowed interference shaping threshold by the primary user.

separation between the secondary users and the interference threshold, i.e., I, allowed by the primary user. These results are in terms of satisfying the interference thresholds at the primary users for 4, 6 and 8 antenna elements. For instance, I = -16dBW is always satisfied with 8 antenna elements, whereas with 6 and 4 antenna elements this interference threshold can be supported for angular separations below 6 and 8 degrees, respectively. The curve without an interference shaping margin, constraint is our benchmark curve. The results for 4 antenna elements show that with angular separations of less than 6 and 8 degrees for interference thresholds of -3dBW, -13dBW and -16dBW, respectively, the users cannot be resolved with finite transmission power. Whereas, with 8 antenna elements all users can be resolved under the interference thresholds. The details are shown in table 3.4.

	4 antenna array	6 antenna array	8 antenna array
AoS for I = 3dBW	5°	5°	5°
AoS for I = -3dBW	6°	5°	5°
AoS for I = -13dBW	6°	6°	5°
AoS for I = -16dBW	8°	6°	5°

Table 3.4: Angel of separation (AoS) using the soft interference shaping strategy for different number of antennas arrays and interference thresholds.

3.5 Conclusion

This chapter addressed the problem of cell-edge user coverage in a primary cellular network by in introducing small cognitive cells in the vicinity of primary cell borders. First a cooperative strategy was developed, where the secondary system can benefit from accessing the spectrum of the primary systems when needed. In return, the secondary BS cooperates with the primary BS by relaying its data towards the primary cell-edge users. Then, a soft interference shaping strategy was introduced, where the interference inflicted on the primary users located outside but within the close vicinity of the cognitive cell borders is controlled within a certain level. The performance of both strategies were evaluated through simulations and, in particular, it was shown that the soft interference shaping strategy demands more power than the cooperative strategy at the secondary BS.

Chapter 4

A Power-Efficient Coverage Scheme for Cell-Edge Users using Cognitive Beamforming

This chapter addresses the problem of strong intercell interference on cell-edge users in conventional cellular networks by deploying cognitive cells within the vicinity of primary cell borders. The cognitive base stations serve primary cell-edge users within the cognitive cells. In return, the cognitive base stations are rewarded by the same spectrum allocated to the primary base stations to serve secondary users. We propose a strategy that is formulated as an optimisation problem for the cognitive cell to minimise the total transmit power of the cognitive base station. This optimisation problem is subjected to maintain a controlled level of interference at the primary outer-cell users falling outside of the cognitive cell and to assure required levels of signal-to-noise-plus-interference-ratio (SINR) at all primary cell-edge and secondary users within the cognitive cell. Simulation results confirm that the beamforming scheme in conjunction with the proposed cognitive structure lead to a significant reduction in overall power transmitted in the network.

4.1 Introduction

Demand for enhancing spectrum reusability and uniform-capacity coverage in cellular networks is growing fast. Although exploiting spatial dimension in multiple antenna wireless systems improves spectrum reuse, intercell interference (ICI) remains a major drawback in uniform improvement of capacity across the cells, particularly at cell boarders. In conventional cellular networks, a major degrading factor affecting the system performance is ICI. This is caused by neighboring cells using the same frequency band, where it can causes significant performance loss to the users, especially the cell-edge users located in the vicinity of the cell boundary. Various techniques have been recommended to mitigate ICI [96], [97]. Users close to the base station (BS) have a high mean SINR, therefore, the focus of this chapter lies in the improvement of the SINR of the cell-edge users. Spectrum utilization improvement, using cognitive radio (CR) has achieved wide acceptance by the wireless community [2]. In a CR network, secondary users are allowed to dynamically access the licensed primary bands. This is provided that primary users in those particular bands are not interfered [1]. Based on satisfaction of certain coexistence constraints in CR systems, generally the secondary users can transmit simultaneously with primary users.

Transmit beamformer design for CR networks at the cognitive BS has been used to control the interference level at the primary users. In this case, each cognitive BS employs beamforming to communicate with the intended secondary user while ensuring that the aggregate interference seen by primary users cannot exceed a specified level [85], [98]. Within this concept, soft interference shaping constraints under the assumption of perfect channel state information (CSI) are introduced in [84] for designing the downlink beamforming vectors so that the ICI leakage on unintended users are kept below some tolerable thresholds. In [65], spatial diversity has been exploited in downlink to improve the channel capacity between secondary users. This was achieved by imposing constraints on the transmit and interference power of secondary and primary users respectively.

In this chapter we consider a multi-cell network, where primary cell-edge users suffer severe ICI due to their location on the cell boundary. As a solution, we explore the problem of ICI mitigation on the primary cell-edge users by deploying cognitive cells at the borders of adjacent primary cells to serve primary cell-edge users. In return, the cognitive cell is rewarded the same spectrum allocated to the primary BSs to transmit to secondary users. In this proposed scenario, it is assumed that the primary outer-cell users, falling outside of the cognitive cell, are served by the primary BS and the secondary/primary cell-edge users are served by the cognitive BS, while simultaneous transmissions to both is maintained. The aim of beamformer design is minimisation of total power at the cognitive BS. This can only be achieved if resource allocation problem is considered as an optimisation problem. This optimisation is subjected to both a SINR constraint on all primary cell-edge and secondary users within the cognitive cell and control of resulting total interference on the other primary outer-cell users located outside of the cognitive cell. It is assumed that the cognitive BS has full CSI of the secondary/primary cell-edge and primary users.

The rest of this chapter is organized as follows. Section 4.2 overviews the proposed system model of the multi-cell cognitive network. In section 4.3, optimisation problem of the proposed system is formulated and presented. Section 4.4, presents the simulation results and evaluate the performance of the proposed scheme. Section 4.5 concludes this chapter.

4.2 System model

This chapter considers a multi-cell network consisting of multiple primary cells and single cognitive cells located on the borders of adjacent primary cells as depicted in Fig. 4.1. Each primary cell has one BS and N primary users within their cells, while

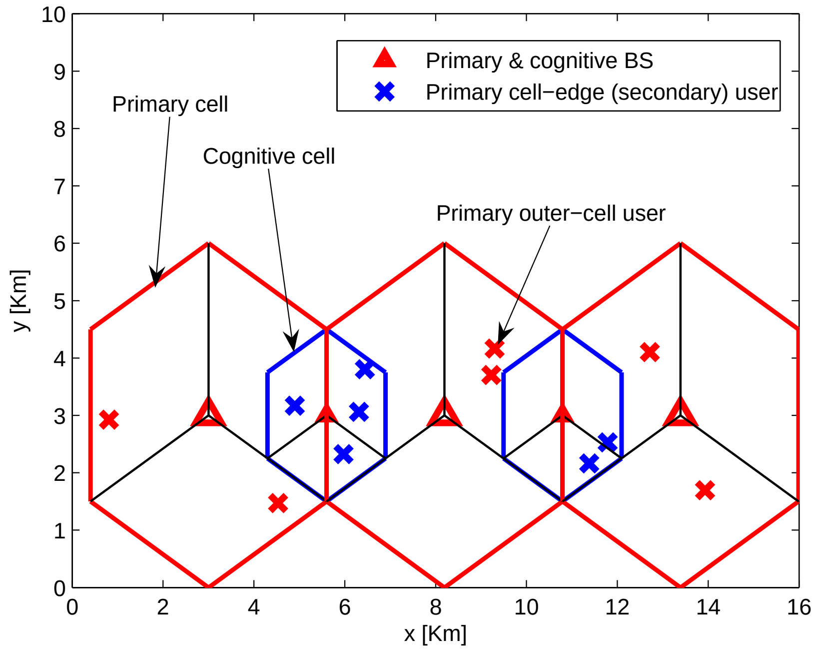

Figure 4.1: Simulation environment, multiple primary cells and single cognitive cells located at the boundaries with multiple primary cell-edge (secondary) and primary outer-cell users.

each cognitive cell has one cognitive BS and K secondary/primary cell-edge users. Every individual cell is divided into three sectors with a BS of linear antenna array of N_t elements per each sector and each user is equipped with a single receive antenna. It is assumed that primary cell-edge users' data are available to the cognitive BSs through reliable backhaul links between the primary and cognitive BSs. The cognitive BS communicates with the secondary users, using the licensed primary spectrum. Note that since any primary cell-edge user located within the cognitive cell are supported by the cognitive BS, therefore, we regard them as secondary users through out this chapter.

The received signal by the secondary user i, i.e., $i \in \{1, 2 \cdots K\}$, is a summation of the intended signal, interference from other secondary users, known as intracell interference, interference from other primary BSs, and background noise which is given by

$$\mathrm{y}_i = \mathbf{h}_{i,s}^H \mathbf{w}_i s_i + \sum_{j=1, j\neq i}^{K} \mathbf{h}_{i,s}^H \mathbf{w}_j s_j + v_i + n_i, \tag{4.1}$$

where $\mathbf{h}_{i,s}^H \in \mathbb{C}^{1\times N_t}$ is the vector channel from the cognitive BS to the i^{th} secondary user, $\mathbf{w}_i \in \mathbb{C}^{N_t\times 1}$ is an associated beamforming vector for secondary user i, s_i is a complex scalar denoting the information signal for secondary user i, v_i indicates any residual ICI caused by all primary BSs on secondary user i, and n_i is the white Gaussian noise at secondary user i. We assume n_i is a zero mean circularly symmetric complex Gaussian (ZMCSCG) random variable with variance σ_i^2, i.e., $\mathrm{n}_i \sim \mathcal{CN}(0, \sigma_i^2)$.

Assuming that the average energy of symbol constellation is normalised to unity, i.e., $\mathbb{E}_{s_i}\left(|s_i|^2\right) = 1$, the SINR secondary user i can be expressed as

$$\mathrm{SINR}_i = \frac{\left|\mathbf{h}_{i,s}^H \mathbf{w}_i\right|^2}{\sum_{j=1, j\neq i}^{K} \left|\mathbf{h}_{i,s}^H \mathbf{w}_j\right|^2 + \xi_i + \sigma_i^2}, 1 \leq i \leq K, \tag{4.2}$$

where $\xi_i = \mathbb{E}(|v_i|^2)$ is the total intercell interference power imposed on secondary user i.

4.3 Cognitive beamforming scheme

We consider an optimisation problem that minimises the total transmitted power by the cognitive BS subject to achieving some desired levels of SINR, denoted as $\gamma_i, i \in \{1, \cdots, K\}$, at the secondary users within the cognitive cell, while avoiding the inflicted interference on the primary outer-cell users, located outside of the cognitive cell, beyond some tolerable levels, denoted as I_m, $m \in \{1, 2 \cdots, N\}$. The problem is formulated as follow

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & \sum_{i=1}^{K} \mathbf{w}_i^H \mathbf{w}_i \\ \text{subject to} \quad & \mathrm{SINR}_i \geq \gamma_i, \quad \forall i \\ & \sum_{n=1}^{K} \left|\mathbf{h}_{m,p}^H \mathbf{w}_n\right|^2 \leq I_m, \quad \forall m \end{aligned} \tag{4.3}$$

where $\mathbf{h}_{m,p}^H \in \mathbb{C}^{1\times N_t}$ is the vector channel from the cognitive BS to the m^{th} primary outer-cell user.

Let

$$\mathbf{H}_p = [\mathbf{h}_{1,p}\ \mathbf{h}_{2,p} \cdots \mathbf{h}_{N,p}]^H, \tag{4.4}$$

$$\mathbf{H}_s = [\mathbf{h}_{1,s}\ \mathbf{h}_{2,s} \cdots \mathbf{h}_{K,s}]^H, \tag{4.5}$$

where $\mathbf{h}_{m,p}^H$ and $\mathbf{h}_{i,s}^H$ indicate the m^{th} and the i^{th} rows of $\mathbf{H}_p$ and $\mathbf{H}_s$ matrices, respectively.

Denoting

$$\mathbf{W} = [\mathbf{w}_1\ \mathbf{w}_2 \cdots \mathbf{w}_K], \tag{4.6}$$

the SINR for secondary user i in (4.2) is rewritten as

$$\mathrm{SINR}_i = \frac{\left|\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_i\right|^2}{\sum_{j=1,j\neq i}^K \left|\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_j\right|^2 + \xi_i + \sigma_i^2}, \tag{4.7}$$

where $\mathbf{e}_i$ and $\mathbf{e}_j$ are the column unit vector with a suitable size which contains all zeros except a one at the i^{th} and j^{th} elements respectively.

Let

$$\beta_i = (\xi_i + \sigma_i^2) \geq 0, \tag{4.8}$$

introducing a slack variable P_o, one can rewrite (4.3) as

$$\begin{aligned} \min_{\mathbf{W},P_0} \quad & P_0 \\ \text{subject to} \quad & \frac{\left|\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_i\right|^2}{\sum_{j=1,j\neq i}^K \left|\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_j\right|^2 + \beta_i} \geq \gamma_i, \quad \forall i \\ & \sum_{n=1}^K \left|\mathbf{e}_m^T\mathbf{H}_p\mathbf{W}\mathbf{e}_n\right|^2 \leq I_m, \quad \forall m \\ & \mathrm{Tr}\left[\mathbf{W}^H\mathbf{W}\right] \leq P_0. \end{aligned} \tag{4.9}$$

The i^{th} SINR constraint in (4.9) can be rearrange as

$$\frac{1}{\gamma_i}\left|\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_i\right|^2 \geq \sum_{j=1,j\neq i}^K \left|\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_j\right|^2 + \beta_i, \tag{4.10}$$

Adding $\left|\mathbf{e}_i^T \mathbf{H}_s \mathbf{W} \mathbf{e}_i\right|^2$ to both sides of (4.10) results in

$$\left(1+\frac{1}{\gamma_i}\right)\left|\mathbf{e}_i^T \mathbf{H}_s \mathbf{W} \mathbf{e}_i\right|^2 \geq \sum_{j=1}^{K}\left|\mathbf{e}_i^T \mathbf{H}_s \mathbf{W} \mathbf{e}_j\right|^2 + \beta_i. \tag{4.11}$$

It can be verified that

$$\sum_{j=1}^{K}\left|\mathbf{e}_i^T \mathbf{H}_s \mathbf{W} \mathbf{e}_j\right|^2 = \left\|(\mathbf{H}_s \mathbf{W})^H \mathbf{e}_i\right\|^2. \tag{4.12}$$

Using (4.12), the SINR constraint in (4.11) is rewritten as follows

$$\left(1+\frac{1}{\gamma_i}\right)\left|\mathbf{e}_i^T \mathbf{H}_s \mathbf{W} \mathbf{e}_i\right|^2 \geq \left\|\begin{bmatrix}(\mathbf{H}_s \mathbf{W})^H \mathbf{e}_i \\ \sqrt{\beta_i}\end{bmatrix}\right\|^2. \tag{4.13}$$

Let $\mathbf{W}^\star$ be an optimal solution to (4.9) and

$$\mathbf{D} = \begin{bmatrix} e^{j\psi_1} & 0 & \cdots & 0 \\ 0 & e^{j\psi_2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & e^{j\psi_K} \end{bmatrix}, \tag{4.14}$$

where ψ_i is an arbitrary phase.

Consider

$$\mathbf{E} = \left(\mathbf{W}^\star \mathbf{D}\right)^H \left(\mathbf{W}^\star \mathbf{D}\right). \tag{4.15}$$

Using (4.6), one can write $\mathbf{E}$ as

$$\mathbf{E} = \begin{bmatrix} \mathbf{w}_1^H \mathbf{w}_1 & \mathbf{w}_1^H \mathbf{w}_2 e^{j(\psi_2-\psi_1)} & \cdots & \mathbf{w}_1^H \mathbf{w}_K e^{j(\psi_K-\psi_1)} \\ \mathbf{w}_2^H \mathbf{w}_1 e^{j(\psi_1-\psi_2)} & \mathbf{w}_2^H \mathbf{w}_2 & \cdots & \mathbf{w}_2^H \mathbf{w}_K e^{j(\psi_K-\psi_2)} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{w}_K^H \mathbf{w}_1 e^{j(\psi_1-\psi_K)} & \mathbf{w}_K^H \mathbf{w}_2 e^{j(\psi_2-\psi_K)} & \cdots & \mathbf{w}_K^H \mathbf{w}_K \end{bmatrix}. \tag{4.16}$$

On the other hand, denoting $\mathbf{F} = \left(\mathbf{W}^\star\right)^H \mathbf{W}^\star$, then

$$\mathbf{F} = \begin{bmatrix} \mathbf{w}_1^H \mathbf{w}_1 & \mathbf{w}_1^H \mathbf{w}_2 & \cdots & \mathbf{w}_1^H \mathbf{w}_K \\ \mathbf{w}_2^H \mathbf{w}_1 & \mathbf{w}_2^H \mathbf{w}_2 & \cdots & \mathbf{w}_2^H \mathbf{w}_K \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{w}_K^H \mathbf{w}_1 & \mathbf{w}_K^H \mathbf{w}_2 & \cdots & \mathbf{w}_K^H \mathbf{w}_K \end{bmatrix}. \tag{4.17}$$

From (4.16) and (4.17), it is clear that $\mathrm{Tr}\left[\mathbf{E}\right] = \mathrm{Tr}\left[\mathbf{F}\right]$. Moreover, plugging $\mathbf{W}^\star \mathbf{D}$ and

$\mathbf{W}^\star$ into (4.7) result in the same value. Therefore, if $\mathbf{W}^\star$ is an optimal solution to the problem (4.9), then $\mathbf{W}^\star\mathbf{D}$ is also an optimal solution. As a result, one can design the beamforming matrix $\mathbf{W}$ up to an arbitrary phase scaling so that the scalar $\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_i$ is always non-negative and real. Then, from (4.13), we can write the SINR constraint in a second-order-cone form as

$$\sqrt{1+\frac{1}{\gamma_i}}(\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_i) \geq \left\| \begin{bmatrix} (\mathbf{H}_s\mathbf{W})^H\,\mathbf{e}_i \\ \sqrt{\beta_i} \end{bmatrix} \right\|. \tag{4.18}$$

Similarly, the interference constraint in (4.9) can be reformulated as

$$\left\|\mathbf{W}^H\mathbf{H}_p^H\mathbf{e}_m\right\| \leq \sqrt{I_m}. \tag{4.19}$$

Finally using vec(.) operator, i.e., vec($\mathbf{A}$) stacks $\mathbf{A}$ into a vector column wise, the left hand side of the power constraint in (4.9) can be rewritten as

$$\mathrm{Tr}\left[\mathbf{W}^H\mathbf{W}\right] = \left\|\mathrm{vec}\left(\mathbf{W}\right)\right\|^2. \tag{4.20}$$

Then, the power constraint in (4.9) is cast in the second-order-cone form as

$$\|\mathrm{vec}(\mathbf{W})\| \leq p_0, \tag{4.21}$$

where $p_0 = \sqrt{P_0}$. Hence, the problem (4.9) can be rewritten as

$$\begin{aligned} \min_{\mathbf{W},p_0} \quad & p_0 \\ \text{subject to} \quad & \sqrt{1+\frac{1}{\gamma_i}}(\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_i) \geq \left\| \begin{bmatrix} (\mathbf{H}_s\mathbf{W})^H\,\mathbf{e}_i \\ \sqrt{\beta_i} \end{bmatrix} \right\|, \\ & \left\|\mathbf{W}^H\mathbf{H}_p^H\mathbf{e}_m\right\| \leq \sqrt{I_m}, \\ & \|\mathrm{vec}(\mathbf{W})\| \leq p_0. \end{aligned} \tag{4.22}$$

Since the scalar constraints in problem (4.22) are conic quadratic inequalities, the problem can be classified as the second order conic programming (SCOP). Applying the Schur complement [43] to the constraints of the resulting SOCP, one can cast the optimisation problem in (4.22) to the standard semidefinite programming (SDP), as follow

$$
\begin{aligned}
\min_{\mathbf{W},p_0} \quad & p_0 \\
\text{subject to} \quad & \mathbf{A} \succeq 0, \quad \forall i \\
& \mathbf{B} \succeq 0, \quad \forall m \\
& \mathbf{C} \succeq 0,
\end{aligned}
\tag{4.23}
$$

where

$$
\begin{aligned}
\mathbf{A} &= \begin{bmatrix} \sqrt{1+\frac{1}{\gamma_i}}\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_i & \left[\mathbf{e}_i^T\mathbf{H}_s\mathbf{W} \quad \sqrt{\beta_i}\right] \\ \begin{bmatrix} \left(\mathbf{W}^H\mathbf{H}_s^H\mathbf{e}_i\right) \\ \sqrt{\beta_i} \end{bmatrix} & \sqrt{1+\frac{1}{\gamma_i}}\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_i\mathbf{I} \end{bmatrix}, \\
\mathbf{B} &= \begin{bmatrix} \sqrt{I_m} & \mathbf{e}_m^T\mathbf{H}_p\mathbf{W} \\ \mathbf{W}^H\mathbf{H}_p^H\mathbf{e}_m & \sqrt{I_m}\mathbf{I} \end{bmatrix}, \\
\mathbf{C} &= \begin{bmatrix} p_0 & \text{vec}^H(\mathbf{W}) \\ \text{vec}(\mathbf{W}) & p_0\mathbf{I} \end{bmatrix}.
\end{aligned}
$$

See Appendix B for the proof.

The problem stated in (4.23) can be solved by optimisation packages, e.g., the SeDuMi solver [42], to attain the precoding matrix $\mathbf{W}$.

4.4 Simulation results

In this section, we compare the performance of the proposed cognitive beamforming scheme with the conventional cellular network.

4.4.1 Simulation setup

We first consider a conventional system with only two primary cells. In this conventional system, one primary BS serves three randomly dropped users, while the other acts as an interfering-primary BS. In the next stage, in order to improve the system performance, a cognitive BS is deployed at the border of primary BSs. In the proposed system, all primary cell-edge users falling within the cognitive cell are served by the cognitive BS while the primary outer-cell users are supported by the primary BS. We obtain a correlated channel model, similar to [54] and [99], by factorizing $\mathbf{h}_{i,s}^H$ as

$$\mathbf{h}_{i,s}^H = \mathbf{h}_w^H \mathbf{R}_{i,s}^{1/2}, \tag{4.24}$$

where $\mathbf{h}_w^H \in \mathbb{C}^{1\times N_t}$ is randomly generated ZMCSCG variables with unit variance, $\mathbf{R}_{i,s} \in \mathbb{C}^{N_t\times N_t}$ is the spatial covariance matrix of user i as seen by the cognitive BS. As shown in [100], the $(q,k)^{th}$ element of the spatial covariance matrix $\mathbf{R}_{i,s}$ in (4.24) is given by

$$\begin{aligned}\mathbf{R}_{i,s}(q,k) =& L_{i,s}\sigma_F^2 e^{-0.5\frac{(\sigma_s \ln 10)^2}{100}} e^{\frac{j2\pi\Delta}{\lambda}[(k-q)\sin\theta_{i,s}]} \\ &\times e^{-2\left[\frac{\pi\Delta\sigma}{\lambda}\{(k-q)\cos\theta_{i,s}\}\right]^2},\end{aligned} \tag{4.25}$$

where $q,k \in [1, N_t]$, $L_{i,s}$ is the path loss coefficient between the cognitive BS and user i, σ_F^2 is the variance of the complex Gaussian fading coefficient between the cognitive BS and user i, σ_s is the standard deviation of the log-normal shadow fading coefficient between the cognitive BS q and user i, i.e., $10^{-\frac{x}{10}}$, $x \sim \mathcal{N}(0, \sigma_s^2)$, Δ is the distance between any two adjacent antenna elements at the cognitive BS, λ is the carrier wavelength, and $\theta_{i,s}$ is the angle of departure for user i with respect to the broadside of the array of the cognitive BS. Furthermore, it is assumed that the angle spread/offset is distributed according to a zero mean normal distribution with standard deviation of σ. Similar method in (4.24) is used to attain the vector channel $\mathbf{h}_{m,p}^H$ from the cognitive BS to the primary outer-cell user m. Simulation parameters are shown in table 4.1. Monte-Carlo simulations are carried out with 5,000 channel realizations per SINR point.

4.4.2 Performance evaluation

Fig. 4.2 displays the total transmit power versus the targeted SINR per user for both conventional and cognitive schemes. In conventional scheme all three users are served by the primary BS, whereas in the proposed cognitive scheme the primary BS only transmits to the primary outer-cell user and the cognitive BS transmits to the remaining two primary cell-edge users such that its inflicted interference on the primary

Table 4.1: Simulation parameters

Parameter	**Value**
Number of antennas per BS	6
Antenna spacing	$\lambda/2$
Array antenna gain	15 dBi
Downlink carrier frequency	1.9 GHz
Noise power spectral density (all users)	-174 dBm/Hz
Noise figure at user receiver	5 dB
Primary cell radius	3 km
Path loss model (l in meters and $l > 35$)	$34.53 + 35\log_{10}(l)$
Angular offset's standard deviation	$2°$
Log-normal shadowing's standard deviation	8 dB
Subchannel bandwidth's wide	15 kHz

outer-cell user is kept under a controlled level denoted by I_m in optimisation problem (4.23). The interference threshold is set at $I_m = 0$ and $I_m = 3\sigma^2$ in this experiment, where σ^2 is the noise variance. Note that the interference imposed on the primary cell-edge users by both primary and interfering-primary BSs have been taken into account in the simulations.

The interference-plus-noise-ratio (INR) at the closest primary cell-edge user to the interfering-primary BS is set at 10 dB. Fig. 4.2 clearly shows that the required power for the conventional scheme is considerably larger than that of the proposed cognitive scheme over the entire range of targeted SINRs.

The same experiment as in Fig. 4.2 is repeated with an INR of 20 dB at the closest primary cell-edge user to the interfering-primary BS when three interference controlling levels of $I_m = 0$, $I_m = 10\sigma^2$ and $I_m = 30\sigma^2$ are considered. The corresponding results are shown in Fig. 4.3. It can be concluded from the results in Figs. 4.2 and 4.3 that increasing the I_m level results in an increase in total transmitting power, due to the fact that the primary BS should increase its transmit power to maintain the required level of SINR at the primary outer-cell user.

Fig. 4.4 illustrates power-saving gain versus the targeted SINR per user for different

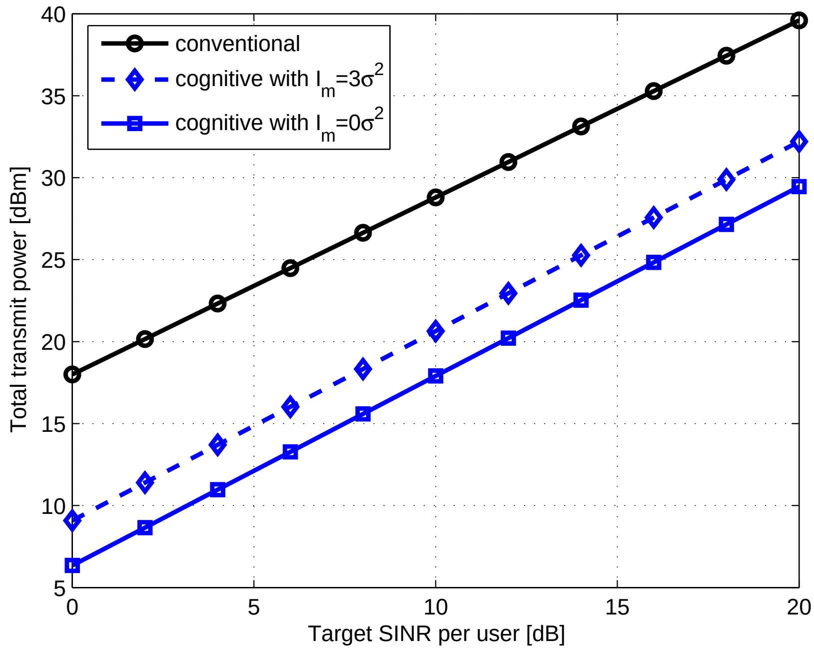

Figure 4.2: Total transmitted power versus the targeted SINR per user when the INR at the closest primary cell-edge user to the interfering-primary BS is set at 10 dB.

values of INR and I_m. The power-saving gain is calculated as $\frac{P_{\text{con}}-P_{\text{cog}}}{P_{\text{con}}} \times 100\%$, where P_{con} and P_{cog} are the total transmit power of the conventional and cognitive schemes, respectively. It can be seen from the figure that the proposed cognitive scheme provides high power-saving gain for all required SINRs. For instance, with $I_m = 0$ and INR=10 dB, the power-saving gain varies from 92.53% to 81.79% when SINR increases from 0 to 20 dB. Fig. 4.4 reveals that when interfering-primary BS increases its transmit power, i.e., resulting in an increased INR at the primary cell-edge users, the proposed cognitive scheme attains higher power-saving gains. For example, with the same value of $I_m = 0$, the proposed approach achieves about 2% higher power-saving gain at INR=20 dB compared with the one at INR=10 dB. It can also be observed from Fig. 4.4 that, under a strong ICI environment the I_m constraint can be relaxed, i.e., increased, while certain power-saving gain can be provided. For instance, at $I_m = 30\sigma^2$ and INR=20 dB, the power-saving gain is still higher than the power-saving gain at $I_m = 3\sigma^2$ and INR=10 dB.

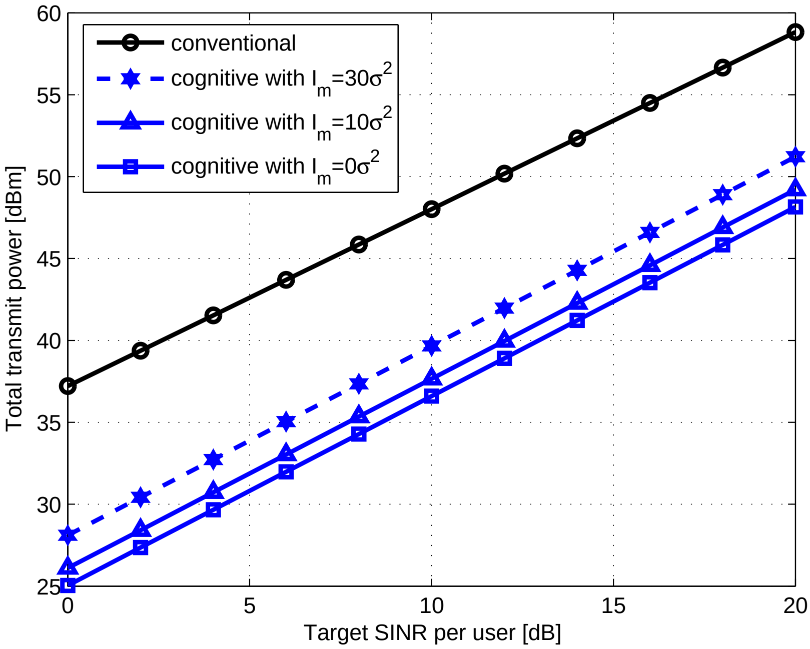

Figure 4.3: Total transmitted power versus the targeted SINR per user when the INR at the closest primary cell-edge user to the interfering-primary BS is set at 20 dB.

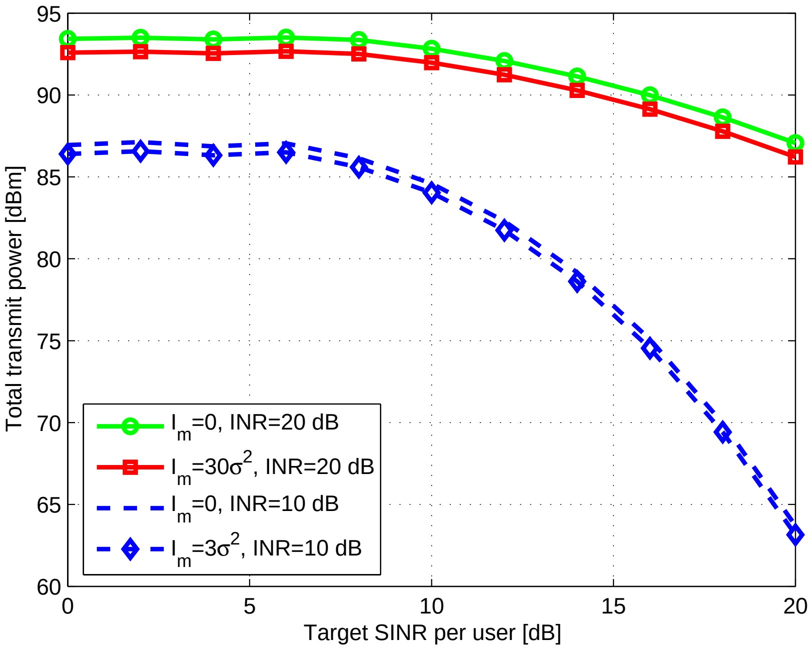

Figure 4.4: Power-saving gain versus the targeted SINR per user with different values of INR and I_m.

4.5 Conclusion

In this chapter the problem of cell-edge user coverage in a multi-cell network is resolved by introducing cognitive cells within the vicinity of primary cells borders. An ICI mitigation method was developed. The method allowed the secondary system to access the spectrum of the primary systems upon need. In return, the cognitive BS supported the primary cell-edge users within the cognitive cell, by treating them as secondary users. In addition, the interference towards the primary users caused by the transmission of cognitive BS was controlled. The performance of the proposed strategy was compared against the conventional cellular network and it was concluded that the conventional method demands more power than the proposed method. Hence, the proposed cognitive scheme outperforms the conventional approach in terms of lower power consumption.

Chapter 5

Robust Cognitive Beamforming for Cell-edge Coverage in Multicell Networks with Probabilistic Constraints

In this chapter, we introduce a downlink beamforming strategy in a cognitive cell located at the boarder of two adjacent cells of a multicell network to support the local cell-edge users of both cells. The proposed strategy is formulated as an optimisation problem to minimise a linear combination of total transmit power of the cognitive base station (BS) and the resulting total interference on the other users located outside of the cognitive cell, so that the signal-to-interference-plus-noise ratio (SINR) targets of the cell-edge users are maintained. In a realistic scenario where CSI may be imperfect, the beamforming design for the cognitive BS based on perfect channel state information (CSI) can easily end up violating the tolerable interference levels of the users falling outside of the cognitive cell. We reformulate the proposed strategy as a robust optimisation problem with outage-probability based constraints to account for the imperfection in CSI. Using the S-Procedure, we transform the intractable probabilistic constraints to a computationally tractable set of conservative deterministic constraints. Finally, applying the rank relaxation, we rewrite the resulting problem in semidefinite programming (SDP) form that can be solved using the standard convex optimisation packages. The simulation results confirm the effectiveness of the proposed robust scheme in power-efficiently expanding the range of achievable SINR targets for the cell-edge users.

5.1 Introduction

Due to the maximal reuse of allocated spectrum in future generation of cellular networks, intercell interference (ICI) remains a fundamental limiting factor for uniform distribution of network capacity across the multiple cells, particularly, in cell boundaries. Network MIMO (multiple-input multiple-output) that brings together multiple antennas of multiple base stations (BSs) via coordination is a central idea in actively exploiting the intercell interference channels for transmitting data stream [8]. This idea has already been considered with very limited leverage of cooperation in LTE-Advanced. In a fully connected network MIMO, multiple BSs across the cellular system need to share data streams of multiple users in order to transmit them jointly and synchronously to the users. However, baseband time synchronization and message sharing among different BSs are currently considered as challenging issues in terms of implementation and inflicting heavy overhead on the backhaul network. On the other hand, in the absence of message sharing, the cellular channel becomes an interference channel if time and spectral resources are to be fully reused in the network.

This chapter explores the problem of ICI mitigation on the users located at the vicinity of cell boarders by deploying a cognitive BS to a cell boundary to serve the cell-edge users. Throughout this chapter, we will call the users who are served only by the main BS of the cell non-cell-edge users to differentiate them from the cell-edge users who are served by the cognitive BS only. It is assumed that the cell-edge users' data are provided to the cognitive BS directly or through the corresponding main BS. While transmitting to the cell-edge users, the deployed BS need to strictly minimise the inflicted interference on the other users of the cell, i.e., the non-cell-edge users. Since such an unselfish way of transmission is the defining feature of cognitive radio [1], [2], we have named the deployed BS as cognitive. By deploying a number of cognitive BSs across the borders or at the vertices of a hexagonal cell, one can effectively isolate the coverage area of the main BS located in the cell center from the BSs of the adjacent cells. However, the problem of interference control on the non-cell-edge users by a cognitive BS becomes a critical matter, particularly, in realistic cellular scenarios where the captured CSI at the cognitive BS is imperfect. It is this problem that we intend to address in this chapter.

Within this concept, soft interference shaping constraints under the assumption of perfect CSI are introduced in [84] for designing the downlink beamforming vectors so that the ICI leakage on unintended users are kept below some tolerable thresholds. In the context of robust cognitive radio and Game theory, the authors in [101] formulate and design a non-cooperative game where a number of secondary users compete with each other over the resources made available by the primary users to maximize their own data rates subject to the transmit power and robust interference constrains. In [102], the authors propose a novel Nash equilibrium [103] and use the variational inequality [104] approach to model and design a concurrent communications of secondary users who coexist with a primary system and compete against each other to maximise their information rate. In game theory, Nash equilibrium is a solution concept of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy unilaterally. The formulation in this chapter is non-robust and contains constraints on the transmit power as well as total interference tolerable by the primary users.

This chapter adopts a different approach based on robust formulation of a cognitive interference management problem, where the CSI between the cognitive BS and the cell-edge or non-cell-edge users are assumed to be imperfect. The imperfection in CSI is due to a number of factors such as inaccurate channel estimation or time delay in feedback channels in realistic cellular scenarios. Hence, the consideration of the underlying channel uncertainties has a paramount amount of importance in developing a cautious beamforming scheme, that ensure the protection of the non-cell-edge users as a result of serving the cell-edge users by the cognitive BS in the proposed scenario. First, we propose a nominal optimisation problem, i.e., based on perfect CSI, for the cognitive BS and then formulate its robust counterpart with imperfect CSI. The proposed problem minimises a linear combination of total downlink transmit power at the cognitive BS and the resulting aggregate interference induced on the non-cell-edge users, i.e., located outside of the area covered by the cognitive BS, subject to assuring certain SINR constraints at the cell-edge users, covered by the cognitive BS.

Earlier studies assume knowledge of channel statistics such as mean and covariance and focus on the average performance without paying attention to the extreme error level. Recently, the extreme case scenario has been considered proportionally by introducing probabilistic constraints on quality of service. When the probabilistic constraint involves linear combination of normally distributed random variables, for example, in most MISO systems, it can be easily reformulated as a convex constraint [105]. Therefore, in our robust formulation, we adopt probabilistic approach based on some adjustable outage probabilities on the SINR constraints and induced interferences on the non-cell-edge users. Using spherical error bounding, rank relaxation and S-procedure, we reformulate the resulting intractable formulation in a numerically tractable robust optimisation problem that can be solved by the SeDuMi [42] solver.

The rest of this chapter is organized as follows. Section 5.2 introduces the system model. Section 5.3 presents the proposed downlink beamforming problem with the assumption of perfect CSI at cognitive BSs, i.e., the non-robust case. In Section 5.4, we reformulate the optimisation problem introduced in Section 5.3 in a probabilistic robust form. Simulation results are presented in section 5.5. Finally, section 5.6 summarises

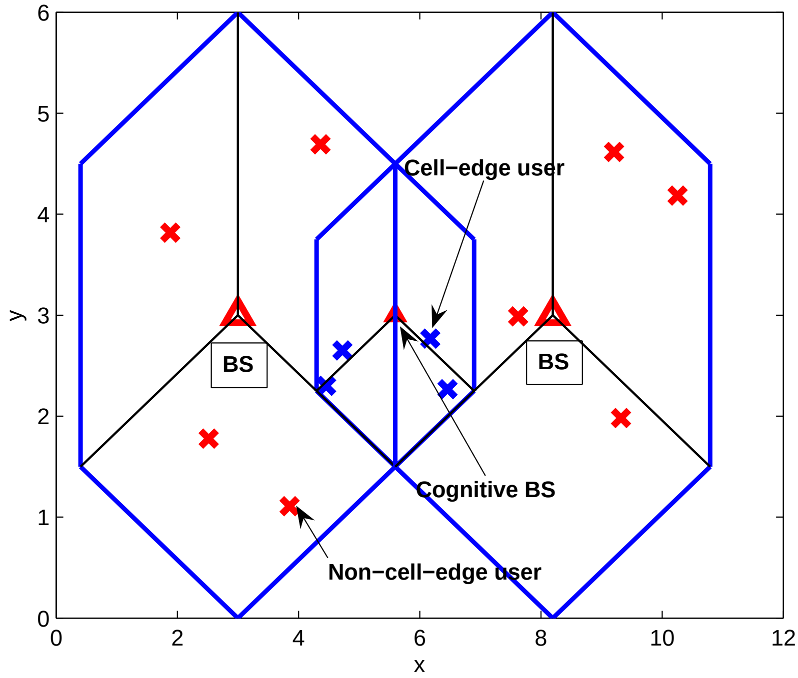

Figure 5.1: A multicell network (x,y in km), consisting of a cognitive cell located on the boundary of the two adjacent cells, and multiple cell-edge and non-cell-edge users.

the chapter with concluding remarks.

5.2 System model

We consider a multicell network consisting of single cognitive cells located on the boundary of adjacent cells, using the same licensed spectrum. As a result, cell-edge users are supported by the cognitive BS. Every individual cell is divided into three sectors with a BS of linear antenna array of M elements per each sector and N single antenna non-cell-edge users, i.e., $\mathcal{S}_o = \{1, \cdots, N\}$. The cognitive cells are also divided into three sectors with a cognitive BS of linear antenna array of M antenna elements and U single antenna cell-edge users, i.e., $\mathcal{S}_l = \{1, \cdots, U\}$ as depicted in Fig. 5.1. It is assumed through out the chapter that each cognitive BS has full CSI of N non-cell-edge and U cell-edge users.

The received signal by the cell-edge user $i \in \mathcal{S}_l$, is a summation of the intended signal, interference from other cell-edge users, known as intracell interference, interference from other BSs and background noise which is given by

$$\mathrm{y}_i = \mathbf{h}_i^H \mathbf{w}_i s_i + \sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{h}_i^H \mathbf{w}_j s_j + v_i + n_i, \tag{5.1}$$

where $\mathbf{h}_i \in \mathbb{C}^M$ is the vector channel from the cognitive BS to the i^{th} cell-edge user, $\mathbf{w}_i \in \mathbb{C}^M$ is an associated beamforming vector for cell-edge user i, s_i is a complex scalar denoting the information signal for cell-edge user i, v_i indicates any residual ICI caused by all BSs on user i, and n_i is the white Gaussian noise at cell-edge user i. We assume n_i is a Zero Mean Circularly Symmetric Complex Gaussian (ZMCSCG) random variable with variance σ_i^2, i.e., $\mathrm{n}_i \sim \mathcal{CN}(0, \sigma_i^2)$.

Assuming that the average energy of symbol constellation is normalized to unity, i.e., $\mathbb{E}_{s_i}\left(|s_i|^2\right) = 1$, the SINR for the cell-edge user i can be expresses as

$$\mathrm{SINR}_i = \frac{\left|\mathbf{h}_i^H \mathbf{w}_i\right|^2}{\sum_{j \in \mathcal{S}_l, j \neq i} \left|\mathbf{h}_i^H \mathbf{w}_j\right|^2 + \xi_i + \sigma_i^2}, \tag{5.2}$$

where $\xi_i = \mathbb{E}(|v_i|^2)$ is the total intercell interference power imposed on cell-edge user i.

5.3 cognitive beamforming strategy with perfect CSI

An optimisation problem is introduced to calculate the downlink beamforming vectors at the cognitive BS, as

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & \sum_{i \in \mathcal{S}_l} \mathbf{w}_i^H \mathbf{w}_i + \sum_{t \in \mathcal{S}_o} \sum_{i \in \mathcal{S}_l} \left|\mathbf{g}_t^H \mathbf{w}_i\right|^2 \\ \text{s. t.} \quad & \frac{\left|\mathbf{h}_i^H \mathbf{w}_i\right|^2}{\sum_{j \in \mathcal{S}_l, j \neq i} \left|\mathbf{h}_i^H \mathbf{w}_j\right|^2 + \xi_i + \sigma_i^2} \geq \gamma_i, \quad \forall i \in \mathcal{S}_l, \end{aligned} \tag{5.3}$$

where γ_i is the SINR level for the cognitive system, and $\mathbf{g}_t \in \mathbb{C}^M$ is the vector channel from the cognitive BS to the t^{th} non-cell-edge user.

It is assumed that the instantaneous fading coefficient vectors, i.e., $\mathbf{h}_i, \forall i \in \mathcal{S}_l$ and

$\mathbf{g}_t, \forall t \in \mathcal{S}_o$, are known at each cognitive BS and any i^{th} cell-edge user, i.e., $i \in \mathcal{S}_l$, can measure the arrived outer-cell interference power ξ_i and report it to the cognitive BS.

The optimisation objective function in (5.3) is the combination of two terms subject to achieving some desired SINR levels, γ_i, for all cell-edge users. In the objective function of (5.3), the first term $\sum_{i\in\mathcal{S}_l} \mathbf{w}_i^H \mathbf{w}_i$ indicates the total transmit power towards cell-edge users within the cognitive cell, whereas, the second term $\sum_{t\in\mathcal{S}_o}\sum_{i\in\mathcal{S}_l} \left|\mathbf{g}_t^H \mathbf{w}_i\right|^2$ represents the overall interference power on the non-cell-edge users due to the transmissions of cell-edge users.

The constraints involve quadratic nonconvex functions of variables. However, it can be modified into the SDP standard formulation. This can be done by changing the vector variables $\mathbf{w}_i$ into matrix variables $\mathbf{W}_i$. Let us define $\mathbf{W}_i = \mathbf{w}_i\mathbf{w}_i^H$, therefore, using the following conditions

$$\mathbf{w}_i^H \mathbf{w}_i = \mathrm{Tr}\left[\mathbf{w}_i \mathbf{w}_i^H\right] = \mathrm{Tr}[\mathbf{W}_i], \tag{5.4}$$

$$\sum_{i\in\mathcal{S}_l} \mathbf{W}_i = \overline{\mathbf{W}}, \tag{5.5}$$

problem (5.3) can be represented as

$$\begin{aligned}
\min_{\mathbf{W}_i} \quad & \sum_{i\in\mathcal{S}_l} \mathrm{Tr}[\mathbf{W}_i] + \sum_{t\in\mathcal{S}_o} \mathrm{Tr}\left[\overline{\mathbf{W}}\mathbf{g}_t\mathbf{g}_t^H\right] \\
\text{s. t.} \quad & \mathrm{Tr}[\mathbf{V}_i\mathbf{h}_i\mathbf{h}_i^H] \geq \xi_i + \sigma_i^2, \\
& \mathbf{W}_i \succeq 0, \\
& \mathrm{rank}(\mathbf{W}_i) = 1, 1 \leq i \leq U,
\end{aligned} \tag{5.6}$$

where

$$\mathbf{V}_i = \frac{\mathbf{W}_i}{\gamma_i} - \sum_{j\in\mathcal{S}_l, j\neq i} \mathbf{W}_j.$$

Problem (5.6) can be solved by the SeDuMi [42] solver to find $\mathbf{W}_i$. However, to obtain the optimal beamforming vectors $\mathbf{w}_i$, $1 \leq i \leq U$, we are only interested in $\mathbf{W}_i$ solutions of (5.6) that are of Rank 1.

5.4 probabilistically-constrained cognitive robust optimisation

The assumption on the beamforming solutions by the previous design is that the cognitive BS has perfect knowledge of $\mathbf{h}_i$ and $\mathbf{g}_t$, i.e., the so-called perfect CSI. This may not achieve the quality of service requirements in the presence of CSI errors. Therefore, beamforming design that take into account these CSI errors are needed.

Using a complex value vector $\mathbf{h}_i \in \mathbb{C}^M$, the channel from cognitive BS to each cell-edge user is determined. This comprises uncertainty in channel gains and is not fully known. The presumed (imperfect) channel vector $\mathbf{h}_i$ can be expressed as

$$\mathbf{h}_i = \hat{\mathbf{h}}_i + \mathbf{e}_i, \tag{5.7}$$

where $\hat{\mathbf{h}}_i \in \mathbb{C}^M$ is the true CSI between cognitive BS and cell-edge user i, and $\mathbf{e}_i \in \mathbb{C}^M$ refers to as the CSI error vector which is assumed to be random. The present formulation in principle concentrates on complex Gaussian CSI errors. It is assumed that $\mathbf{e}_i \sim \mathcal{CN}(0, \mathbf{C}_i)$, where $\mathbf{C}_i \succeq 0, i \in \mathcal{S}_l$ is known as the error covariance matrix.

The same implies to the estimate CSI between the cognitive BS and non-cell-edge users noted as

$$\mathbf{g}_t = \hat{\mathbf{g}}_t + \mathbf{e}_t, \tag{5.8}$$

where $\hat{\mathbf{g}}_t \in \mathbb{C}^M$ is the true CSI between cognitive BS and non-cell-edge user t, and $\mathbf{e}_t \in \mathbb{C}^M$ refers to as CSI error vector. It is assumed that $\mathbf{e}_t \sim \mathcal{CN}(0, \mathbf{C}_t)$, where $\mathbf{C}_t \succeq 0, i \in \mathcal{S}_o$ is known as the error covariance matrix.

In the robust design problem the SINR expression for the cell-edge user i can be written as

$$\text{SINR}_i = \frac{\mathbf{w}_i^H(\hat{\mathbf{h}}_i + \mathbf{e}_i)(\hat{\mathbf{h}}_i + \mathbf{e}_i)^H\mathbf{w}_i}{\sum_{j\in\mathcal{S}_l, j\neq i}\mathbf{w}_j^H(\hat{\mathbf{h}}_i + \mathbf{e}_i)(\hat{\mathbf{h}}_i + \mathbf{e}_i)^H\mathbf{w}_j + \xi_i + \sigma_i^2}. \tag{5.9}$$

The optimisation problem in (5.3) can be reformulated as a robust beamforming design which is constrained by probabilistic SINR as follow

$$
\begin{aligned}
\min_{\mathbf{w}_i} \max_{\mathbf{e}_t} \quad & \sum_{i\in\mathcal{S}_l} \mathbf{w}_i^H \mathbf{w}_i + \sum_{t\in\mathcal{S}_o}\sum_{i\in\mathcal{S}_l} \mathbf{w}_i^H (\hat{\mathbf{g}}_t + \mathbf{e}_t)(\hat{\mathbf{g}}_t + \mathbf{e}_t)^H \mathbf{w}_i \\
\text{s. t.} \quad & \Pr\left\{\min_{\mathbf{e}_i} \quad \mathrm{SINR}_i \geq \gamma_i\right\} \geq 1-\rho_i,
\end{aligned} \tag{5.10}
$$

where Pr(.) is the probability, and $\rho_i \in (0,1]$ is a preselected value. The robust beamforming problem in (5.10) guarantees that for all admissible channel errors, each user is served with an SINR not smaller than γ_i at least $(1-\rho_i)\times 100\%$ of the period.

As shown in problem (5.10), individual cell-edge user i in the SINR inequality constraint does not yield simple closed form expressions for the considered CSI error distribution models. Let us define the CSI errors as $\mathbf{e}_i = \mathbf{C}_i^{\frac{1}{2}}\hat{\mathbf{e}}_i$, and $\mathbf{e}_t = \mathbf{C}_t^{\frac{1}{2}}\hat{\mathbf{e}}_t$, where $\hat{\mathbf{e}}_i \sim \mathcal{CN}(0, \mathbf{I}_M)$, and $\hat{\mathbf{e}}_t \sim \mathcal{CN}(0, \mathbf{I}_M)$. By introducing slack variable k, and substituting for $\mathbf{e}_i$, and $\mathbf{e}_t$, one can rewrite problem (5.10) as

$$
\begin{aligned}
\min_{\mathbf{W}_i,\mathrm{k}} \quad & \mathrm{k} + \sum_{i\in\mathcal{S}_l} \mathrm{Tr}\left(\mathbf{W}_i\right) \\
\text{s. t.} \quad & \Pr\left\{(\hat{\mathbf{h}}_i + \mathbf{C}_i^{\frac{1}{2}}\hat{\mathbf{e}}_i)^H \mathbf{V}_i (\hat{\mathbf{h}}_i + \mathbf{C}_i^{\frac{1}{2}}\hat{\mathbf{e}}_i) \geq \xi_i + \sigma_i^2\right\} \geq 1-\rho_i, \\
& \Pr\left\{\sum_{t\in\mathcal{S}_o}\sum_{i\in\mathcal{S}_l} (\hat{\mathbf{g}}_t + \mathbf{C}_t^{\frac{1}{2}}\hat{\mathbf{e}}_t)^H \mathbf{W}_i (\hat{\mathbf{g}}_t + \mathbf{C}_t^{\frac{1}{2}}\hat{\mathbf{e}}_t) \leq \mathrm{k}\right\} \geq 1-\rho_t, \\
& \mathbf{W}_i \succeq 0, \\
& \mathrm{rank}(\mathbf{W}_i) = 1, \ 1 \leq i \leq U,
\end{aligned} \tag{5.11}
$$

where $\rho_t \in (0,1]$ is a preselected value, making sure that the interference towards each non-cell-edge user is no greater than a threshold at least $(1-\rho_t)\times 100\%$ of the time.

Except for the rank-one constraint, i.e., $\mathrm{rank}(\mathbf{W}_i) = 1$, the remaining problem is convex. By dropping this constraint, we obtain a relaxed SDP problem [57], which is convex and far easier to solve. The major issue associated with removal of the $\mathrm{rank}(\mathbf{W}_i) = 1$ is in relation with having rank higher than one of solution in problem (5.11). The problem can be solved using conservative step, where finding a convex approximation of (5.11) is targeted.

Conservative Reformulation

The conservative Reformulation aims to find a convex approximation of (5.11), in a conservative (restrictive) sense. In the sequel, we use S-Procedure [43], to rewrite the constraints in (5.11) that involve quadratic inequalities in error vectors in the linear matrix inequality forms.

Lemma 5.1 (The S-Procedure [43]). *Let*

$$f_i(\mathbf{x}_i) = \mathbf{x}_i^H \mathbf{A}_i \mathbf{x}_i + 2Re\left\{\mathbf{x}_i^H \mathbf{b}_i\right\} + c_i, \quad \text{for } i = 1, 2, \tag{5.12}$$

where $\mathbf{A}_i \in \mathbb{H}^M, \mathbf{b}_i \in \mathbb{C}^M$, *and* $c_i \in \mathbb{R}$. *Suppose that there exists an* $\hat{\mathbf{x}}_i \in \mathbb{C}^M$ *such that* $f_2(\hat{\mathbf{x}}_i) < 0$. *Then for all* $\mathbf{x}_i \in \mathbb{C}^M$, *the following two conditions are equivalent*

1. $f_1(\mathbf{x}_i) \geq 0$ *and* $f_2(\mathbf{x}_i) \leq 0$ *are satisfied for all* $\mathbf{x}_i$,

2. *There exists a* $\lambda \geq 0$ *such that*

$$\begin{bmatrix} \mathbf{A}_1 + \lambda \mathbf{A}_2 & \mathbf{b}_1 + \lambda \mathbf{b}_2 \\ \mathbf{b}_1^H + \lambda \mathbf{b}_2^H & c_1 + \lambda c_2 \end{bmatrix} \succeq 0.$$

Suppose that we have two sets $\upsilon \subset \mathbb{C}^M$, and $\beta \subset \mathbb{C}^M$, where υ, and β are certain subsets of all the possible values of $\hat{\mathbf{e}}_i$, and $\hat{\mathbf{e}}_t$, respectively. For an M-dimensional complex vector these subsets might have weird shapes from a geometrical point of view. Therefore, υ, and β are chosen as a spherical sets such that

$$\begin{aligned} \upsilon &= \left\{\hat{\mathbf{e}}_i \in \mathbb{C}^M | \, \|\hat{\mathbf{e}}_i\|^2 \leq \mathrm{d}_i{}^2\right\}, \\ \beta &= \left\{\hat{\mathbf{e}}_t \in \mathbb{C}^M | \, \|\hat{\mathbf{e}}_t\|^2 \leq \mathrm{d}_t{}^2\right\}, \end{aligned} \tag{5.13}$$

where d_i, and d_t are the radius of υ, and β spheres, respectively.

Using (5.13), problem (5.11) can be rewritten with constraints as the same form as

(5.12) shown as

$$\begin{aligned}
\min_{\mathbf{W}_i,\mathrm{k}} \quad & \mathrm{k} + \sum_{i\in\mathcal{S}_l} \mathrm{Tr}\left(\mathbf{W}_i\right) \\
\text{s. t.} \quad & \Pr\{\mathrm{f}_1^{[1]}(\hat{\mathbf{e}}_i)\} \geq 1-\rho_i, \\
& \mathrm{f}_2^{[1]}(\hat{\mathbf{e}}_i) = \hat{\mathbf{e}}_i{}^H \mathrm{I}_M \hat{\mathbf{e}}_i - \mathrm{d}_i{}^2 \leq 0, \\
& \Pr\{\mathrm{f}_1^{[2]}(\hat{\mathbf{e}}_t)\} \geq 1-\rho_t, \\
& \mathrm{f}_2^{[2]}(\hat{\mathbf{e}}_t) = \hat{\mathbf{e}}_t{}^H \mathrm{I}_M \hat{\mathbf{e}}_t - \mathrm{d}_t{}^2 \leq 0, \\
& \mathbf{W}_i \succeq 0,
\end{aligned} \tag{5.14}$$

where

$$\mathrm{f}_1^{[1]}(\hat{\mathbf{e}}_i) = \hat{\mathbf{h}_i}^H \mathbf{V}_i \hat{\mathbf{h}_i} + \hat{\mathbf{h}_i}^H \mathbf{V}_i \mathbf{C}_i^{\frac{1}{2}} \hat{\mathbf{e}}_i + \hat{\mathbf{e}}_i{}^H \mathbf{C}_i^{\frac{1}{2}} \mathbf{V}_i \hat{\mathbf{h}_i} + \hat{\mathbf{e}}_i{}^H \mathbf{C}_i^{\frac{1}{2}} \mathbf{V}_i \mathbf{C}_i^{\frac{1}{2}} \hat{\mathbf{e}}_i - \xi_i - \sigma_i^2 \geq 0,$$

$$\mathrm{f}_1^{[2]}(\hat{\mathbf{e}}_t) = \sum_{t\in\mathcal{S}_o}\sum_{i\in\mathcal{S}_l}(-\hat{\mathbf{g}}_t{}^H \mathbf{W}_i \hat{\mathbf{g}}_t - \hat{\mathbf{g}}_t{}^H \mathbf{W}_i \mathbf{C}_t^{\frac{1}{2}} \hat{\mathbf{e}}_t - \hat{\mathbf{e}}_t{}^H \mathbf{C}_t^{\frac{1}{2}} \mathbf{W}_i \hat{\mathbf{g}}_t - \hat{\mathbf{e}}_t{}^H \mathbf{C}_t^{\frac{1}{2}} \mathbf{W}_i \mathbf{C}_t^{\frac{1}{2}} \hat{\mathbf{e}}_t + \mathrm{k}) \geq 0.$$

Furthermore

$$\mathrm{f}_1^{[1]}(\hat{\mathbf{e}}_i) = \hat{\mathbf{e}}_i{}^H \mathbf{C}_i^{\frac{1}{2}} \mathbf{V}_i \mathbf{C}_i^{\frac{1}{2}} \hat{\mathbf{e}}_i + 2\mathrm{Re}\{\hat{\mathbf{e}}_i{}^H \mathbf{C}_i^{\frac{1}{2}} \mathbf{V}_i \hat{\mathbf{h}_i}\} + \hat{\mathbf{h}_i}^H \mathbf{V}_i \hat{\mathbf{h}_i} - \xi_i - \sigma_i^2 \geq 0,$$

$$\mathrm{f}_1^{[2]}(\hat{\mathbf{e}}_t) = \sum_{t\in\mathcal{S}_o}\sum_{i\in\mathcal{S}_l}(-\hat{\mathbf{e}}_t{}^H \mathbf{C}_t^{\frac{1}{2}} \mathbf{W}_i \mathbf{C}_t^{\frac{1}{2}} \hat{\mathbf{e}}_t - 2\mathrm{Re}\{\hat{\mathbf{e}}_t{}^H \mathbf{C}_t^{\frac{1}{2}} \mathbf{W}_i \hat{\mathbf{g}}_t\} - \hat{\mathbf{g}}_t{}^H \mathbf{W}_i \hat{\mathbf{g}}_t + \mathrm{k}) \geq 0.$$

Due to the Gaussian distribution of the error vectors, $\hat{\mathbf{e}}_i$ and $\hat{\mathbf{e}}_t$ may fallout of their relevant subset. In that case, we will experience SINR and interference outages. It is assumed that this design will always tolerate a certain outage. Problem (5.14) will hold if

$$\begin{aligned}
\Pr\left\{\hat{\mathbf{e}}_i \in \upsilon\right\} &\geq 1-\rho_i, \\
\Pr\left\{\hat{\mathbf{e}}_t \in \beta\right\} &\geq 1-\rho_t.
\end{aligned} \tag{5.15}$$

It can be shown that by choosing d_i and d_t as

$$\begin{aligned}
\mathrm{d}_i &= \sqrt{\frac{\mathrm{ICDF}_m(1-\rho_i)}{2}}, \\
\mathrm{d}_t &= \sqrt{\frac{\mathrm{ICDF}_m(1-\rho_t)}{2}},
\end{aligned} \tag{5.16}$$

where ICDF(.) is the inverse cumulative distribution function of the Chi-square random variable with m degree of freedom, problem (5.14) is satisfied.

Applying Lemma 1 enables us to turn the infinitely many constraints in (5.14) into a set of traceable constraints. Therefore, we can transform problem (5.14) to a convex form as shown in (5.17), which will be our robust beamforming solution.

$$
\begin{aligned}
\min_{\mathbf{W}_i,\mathrm{k}} \quad & \mathrm{k} + \sum_{i\in\mathcal{S}_l} \mathrm{Tr}\left(\mathbf{W}_i\right) \\
\text{s. t.} \quad & \\
& \begin{bmatrix} \mathbf{C}_i^{\frac{1}{2}}\mathbf{V}_i\mathbf{C}_i^{\frac{1}{2}} + \lambda_i \mathrm{I}_M & \mathbf{C}_i^{\frac{1}{2}}\mathbf{V}_i\hat{\mathbf{h}}_i \\ \hat{\mathbf{h}}_i^{H}\mathbf{V}_i\mathbf{C}_i^{\frac{1}{2}} & \hat{\mathbf{h}}_i^{H}\mathbf{V}_i\hat{\mathbf{h}}_i - \xi_i - \sigma_i^2 - \lambda_i \mathrm{d}_i^2 \end{bmatrix} \succeq 0, \\
& \begin{bmatrix} -\sum_{t\in\mathcal{S}_o}\sum_{i\in\mathcal{S}_l} \mathbf{C}_t^{\frac{1}{2}}\mathbf{W}_i\mathbf{C}_t^{\frac{1}{2}} + \lambda_t \mathrm{I}_M & -\sum_{t\in\mathcal{S}_o}\sum_{i\in\mathcal{S}_l} \mathbf{C}_t^{\frac{1}{2}}\mathbf{W}_i\hat{\mathbf{g}}_t \\ -\sum_{t\in\mathcal{S}_o}\sum_{i\in\mathcal{S}_l} \hat{\mathbf{g}}_t^{H}\mathbf{W}_i\mathbf{C}_t^{\frac{1}{2}} & -\sum_{t\in\mathcal{S}_o}\sum_{i\in\mathcal{S}_l} \hat{\mathbf{g}}_t^{H}\mathbf{W}_i\hat{\mathbf{g}}_t + \mathrm{k} - \lambda_t \mathrm{d}_t^2 \end{bmatrix} \succeq 0, \\
& \lambda_i \geq 0, \\
& \lambda_t \geq 0, \\
& \mathbf{W}_i \succeq 0.
\end{aligned}
\tag{5.17}
$$

5.5 simulation results

This section presents some numerical examples illustrating the performances of our proposed schemes and finally compared together. For simplicity, the scenario is assumed with one BS serving one non-cell-edge user and one cognitive BS serving two cell-edge users (i.e., $N = 1$, $M = 2$). cognitive and other BSs are allocated with an array of 6 antenna elements (i.e., $M = 6$). The elements of the channel from the cognitive BS to either non-cell-edge or cell-edge users are assumed independent and identically distributed (i.i.d.) complex Gaussian distributed with mean 0 and variance 1.

Monte-Carlo simulations are carried out over 20 independent user distributions with 2000 channel realizations, where each user distribution consists of 3 randomly

Table 5.1: Simulation parameters

Parameter	**Value**
Number of antennas per BS	6
Antenna spacing	$\lambda/2$
Array antenna gain	15 dBi
Downlink carrier frequency	1.9 GHz
Noise power spectral density (all users)	-174 dBm/Hz
Noise figure at user receiver	5 dB
cell radius	3 km
Path loss model ($l > 35$ in meter)	$34.53 + 38\log_{10}(l)$
Angular offset's standard deviation	$2°$
Log-normal shadowing's standard deviation	8 dB

located users in the network. We consider the effect of small-scale fading caused by two antennas separated by a fractional of a meter and also large-scale fading caused by the shadowing conditions. Therefore, we have used the following model [106] for the channel model setup

$$\mathbf{h}_i = 10^{-(34.53+38\log_{10}(l))/2}.\psi_i.\varphi_i.(\hat{\mathbf{h}_i} + \hat{\mathbf{e}}_i), \tag{5.18}$$

where l is the distance between the cognitive BS and the i^{th} cell-edge user, ψ_i is the shadowing, φ_i is the antenna gain. The CSI errors are spatially i.i.d. and have standard complex Gaussian distributions; i.e., $\mathbf{C}_i = \mathbf{C}_t = \sigma_e^2 \mathrm{I}_M$. The outage probability requirements are set to $\rho_i = \rho_t = \rho$. The rest of the parameters, which are based on the LTE standard are shown in table 5.1. In Figs. 5.2 and 5.3 we draw the total transmit power of the network versus various SINR targets at user terminals for the proposed scheme, for different values of σ_e. We can observe from Figs. 5.2 and 5.3 that the robust conventional scheme, where there is no control on the interference, i.e., the second utility is removed in (5.10), fails to operate efficiently in terms of power consumption beyond 12 dB and 8 dB, respectively, of SINR target. This is due to the fact that the cognitive BS keep increasing its transmit power to maintain the SINR requirements of the cell-edge users and, inevitably, keeps increasing its interference on the non-cell-edge users. Whereas the second utility of the objective function of the proposed optimisation problem in (5.10) controls the inflicted interference by the cognitive BS and stabilizes the egoistic dynamic of the robust conventional network in an equilibrium point, agreed by all BSs. The robust and non–robust design problems outperform the robust conventional method in terms of transmit power. For σ_e=0.01, the proposed robust method provides a very tight bound. It is observed that for each ρ, there exists a critical SINR beyond which the optimisation problem becomes infeasible. For instance in Fig. 5.3, 18 dB is the critical SINR for $\sigma_e = 0.05$, and ρ= 0.1. Since the power required with $\sigma_e = 0.05$ is only 2 dBm more than the non-robust case for a majority of feasible SINR values, the proposed method provides tight bounds for relatively small values of σ_e, that are reasonable in a practical scenario. These figures show that at a given SINR, the total transmit power increases as the uncertainty level increases. The results also confirm that achieving robustness at higher uncertainty levels comes at the expense of lower achievable limits of SINR targets at affordable power levels.

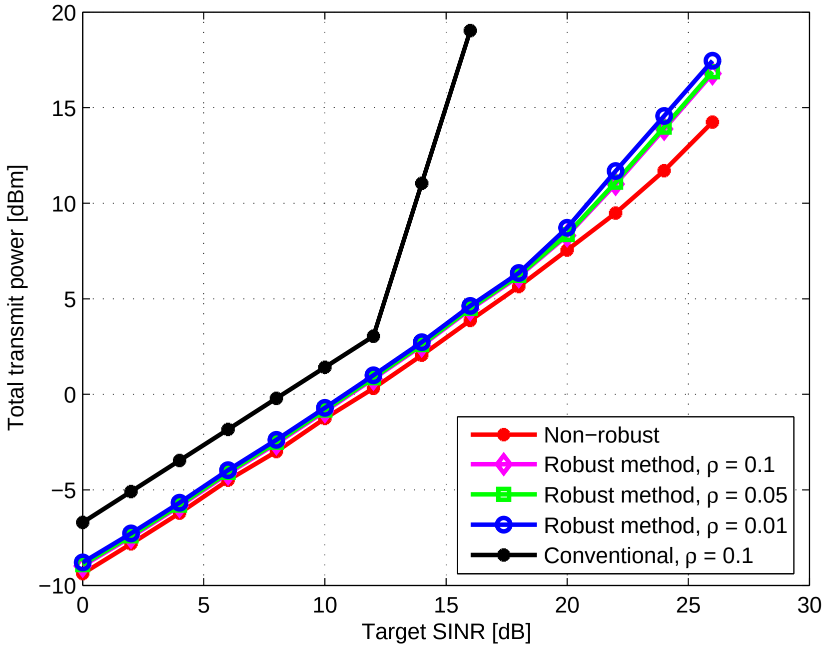

Figure 5.2: Total transmit power versus targeted SINR for the proposed and robust conventional schemes when σ_e=0.01.

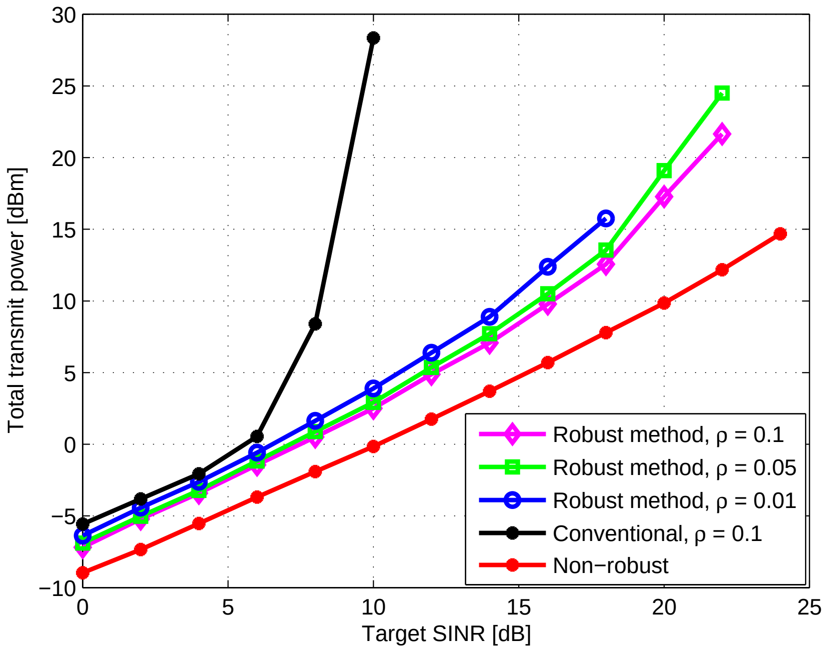

Figure 5.3: Total transmit power versus targeted SINR for the proposed and robust conventional schemes when σ_e=0.05.

5.6 conclusion

A downlink beamforming strategy for a mutilcell network with perfect CSI at a cognitive BS was proposed. The design was based on allocating a cognitive cell at the boundary of two adjacent cells and, therefore, supporting the cell-edge users within the cognitive cell. The aim was to minimise the linear combination of the total downlink transmit power at the cognitive BS and the interference towards the non-cell-edge users subject to SINR targets of the cell-edge users. In the next step, the problem was reformulated as a robust optimisation problem with outage-probability based constraints to account for the imperfection in CSI. Application of the relaxation of the rank constraint and the S-Procedure led to a transformation of the optimisation problem into SDP. Simulation results were carried out and the proposed robust method was compared with the non-robust and the conventional cases. The proposed robust design outperformed the robust conventional scheme in terms of minimised transmit power. The results confirmed that increases in the uncertainty region of the CSI and the outage probabilities not only lead to increased power consumption at the cognitive BS, but also adversely affect the range of quality of service in term of limited SINR targets, achievable at affordable levels of power consumption at the cognitive BS.

Chapter 6

Conclusions and future work

Optimising the network performance and calibrating the efficiency of various resource allocation schemes while ignoring the impact of intercell interference (ICI) may be highly misleading for practical scenarios. Therefore, it is of immense importance for the system designers to develop reasonable bounds while calibrating the efficiency of a variety of resource allocation schemes available in the literature.

Cognitive radio can become an important enabling technology to take advantage of the free resources in the licensed spectrum. Likewise, the technology increases the spectrum efficiency by transmitting data when the licensed (primary) users leave some free channel. This thesis is framed within the field of cognitive radio, a smarter communications paradigm in which radios may learn and adapt to the environment. While this novel scheme promises a better spectrum utilisation by allowing dynamic access in certain primary bands, there exists a series of challenges, such as interference management towards the primary system, which need to be addressed before the technology is mature enough for its deployment. The quantitative and performance criteria are one of the most important strategies in interference management in a cognitive radio system. They offer a set of techniques and strategies to evaluate the performance of the coexistence between the primary and the unlicensed (secondary) systems, while respecting the regulatory policies and constraints.

The aim of this thesis is to reduce the overall power consumption of the cognitive cellular network while ensuring required levels of signal-to-interference-plus-noise ratios for all user terminals located within the cognitive cell. As a result, primary cell-edge users get supported by the cognitive base station and the problem of ICI is resolved. In return, the cognitive base station can serve secondary users. Interference has been identified as a challenge to be tackled in order to achieve this goal. To this end, this thesis exploited problems in the area of beamforming for cognitive cellular networks and proposed several interference management techniques based on beamforming. We also have examined the robust downlink beamforming technique in a multi-user MISO cognitive cellular network, where the CSI is assumed to be imperfectly known and is impaired by an ambient uncertainty. For the different scenarios, the optimal solution of the problem was investigated and low complexity efficient algorithms were proposed. Furthermore, the impact of the different constraints was studied.

Based on practical assumptions, the proposed cognitive beamformer designs in this thesis can provide spectrum efficiency, higher data rate, support for the cell-edge users distant form their base stations, and efficiency in terms of power consumption. However there are some cost for deploying cognitive base stations on the primary cells borders and the complexity with high number of users within the network.

This chapter summarises the findings of previous chapters and outlines possible future research directions.

6.1 Thesis summary

The introductory chapter outlined the motivation of this thesis and defined the open issues regarding interference management in multi-cell cognitive networks. The contributions of this thesis were also stated.

6.1.1 Summary of Chapter 2

This chapter reviewed the concepts of convex and robust optimisations. Furthermore, we reviewed the principles of beamforming using linear antenna array along with concepts of second order cone programming and semidefinite programming. An optimisation problem was presented to calculate transmit beamformers for multiple active users in a single-cell scenario. Different approaches to solve the optimisation problem were outlined.

6.1.2 Summary of Chapter 3

In this chapter, a multi-cell beamforming scheme was proposed. We focused on downlink cognitive communication within a cellular network. A cluster of primary cells sharing the same bandwidth was considered. In addition, small cognitive cells were located within the the primary cells boundaries to support both primary cell-edge and secondary users. First, a cooperative strategy was introduced where the secondary BS relays data, received from the primary BS, to a primary cell-edge user, while transmitting data to the secondary users within the cognitive cell, through spatial multiplexing. Second, a soft interference shaping strategy was proposed where the secondary BS could focus its radiation pattern along the direction of all users (primary cell-edge/secondary) within the cognitive cell, while forming nulls with controlled depths towards the primary users located outside but within the close vicinity of the cognitive cell border. In other words, the primary users surrounding the cognitive cell border did not tolerate interference level above an allowed threshold from the secondary system. The optimisation problem was formulated in the standard Semidefinite programming. An iterative algorithm was developed to find optimal solution to the optimisation problem of the multi-cell cognitive beamforming scheme. The convergence of the algorithm depended on the number of antenna elements, the targeted SINRs, and the number of active users. The performance of both strategies were evaluated through simulations and, in particular, it was shown that the soft interference shaping strategy demands more power than the cooperative strategy at the secondary BS.

6.1.3 Summary of Chapter 4

In this chapter we considered a multi-cell network, where primary cell-edge users suffer severe ICI due to their location on the cell boundary. Therefore, we managed the problem of ICI mitigation on the primary cell-edge users by deploying cognitive cells at the borders of adjacent primary cells to serve primary cell-edge users. In return, the cognitive cell was rewarded the same spectrum allocated to the primary BSs to transmit to secondary users. In this proposed scenario, it was assumed that the primary outer-cell users, falling outside of the cognitive cell, were served by the primary BS and the secondary/primary cell-edge users were served by the cognitive BS, while simultaneous transmissions to both was maintained. The aim of beamformer design was the minimisation of total power at the cognitive BS. This can only be achieved if resource allocation problem was considered as an optimisation problem. This optimisation was subjected to both a SINR constraint on all primary cell-edge and secondary users within the cognitive cell and control of resulting total interference on the other primary outer-cell users located outside of the cognitive cell. It was assumed that the cognitive BS had full CSI of the secondary/primary cell-edge and primary users. The optimisation problem for the proposed cognitive beamforming using instantaneous CSI was formulated in standard semidefinite programming form. The performance of the proposed strategy was compared against the conventional cellular network and it was concluded that the conventional method demands more power than the proposed method. Hence, the proposed cognitive scheme outperforms the conventional approach in terms of lower power consumption.

6.1.4 Summary of Chapter 5

In this chapter, a beamforming strategy for downlink transmission in multi-cell cognitive networks was proposed. Moreover, the iterative algorithm proposed for this strategy was the solution to an intercell interference balancing optimisation problem. This problem minimised a linear combination of data transmission power at each cognitive BS and the resulting interference power (caused by the the cognitive transmission) towards each cell-edge user, while maintaining the required SINRs by the all users within the cognitive cell. We first proposed this different approach based on formulation of a cognitive interference management problem, where the CSI between the cognitive BS and the cell-edge or non-cell-edge users were assumed to be imperfect. We then reformulated the proposed strategy as a robust optimisation problem with imperfect CSI for the cognitive BS with outage-probability based constraints. Using the S-Procedure, we transformed the intractable probabilistic constraints to a computationally tractable set of conservative deterministic constraints. Finally, applying the rank relaxation, we rewrote the resulting problem in semidefinite programming. Simulation results were carried out and the proposed robust method was compared with the non-robust and the conventional cases. The proposed robust design outperformed the robust conventional scheme in terms of minimised transmit power. The results confirmed that increases in the uncertainty region of the CSI and the outage probabilities not only lead to increased power consumption at the cognitive BS, but also adversely affect the range of quality of service in term of limited SINR targets, achievable at affordable levels of power consumption at the cognitive BS.

6.2 Future research directions

The contributions of this thesis suggest the following future research directions related to beamforming techniques for multi-cell cogniitve processing.

6.2.1 Robust beamforming

In downlink multi-cell networks, channel state information is required to design transmit beamforming/pre-coding vectors for all users terminals. This increases the burden on signaling overhead between BSs and their user terminals, especially for a large size of coordinating BSs. For this reason, algorithms that demand less signaling overhead are desirable for multi-cell processing. Furthermore, the obtained channel state information at the BS, i.e., the CSIT, may not be accurate due to the channel estimation error. As the system design based on corrupt CSIT may not function as expected in the realistic channel conditions, robust schemes to the uncertainties in CSIT are also of interests.

In current treatment of the robust beamforming, we usually start from a SOCP problem formulation, and using S-Procedure based methods, we encounter with a SDP. But there is another way of treating these problems proposed by Bertsimas & Sim [107]. In this treatment, the robust counterpart of any problem exhibits the same structure, i.e., the robust counterpart of SOCP based beamforming problems are also SOCPs with more constraints and variables. It is recommended to assess the performance of this new treatment as well.

6.2.2 Rate maximisation under power constraint

The focus of this thesis is on energy efficiency. Therefore, the objective functions defined in Chapters 3 and 4 are to minimise total transmit power across cognitive BSs. On the other hand, the objective function introduced in Chapter 5 is to minimise a linear combination of two utility functions, characterising each BS's weighted sum of transmitted power to the cognitive/cell-edge users and its resulting weighted sum of interference power inflicted upon the outer-cell users of the other cells. The constraints of all optimisation problems introduced in this thesis are on users' signal-to-interference-plus-noise ratios (SINRs). In other words, beamforming schemes proposed in this thesis ensure all users' quality of services above requirement levels with minimum total transmit power. A possible extension for the work in this thesis is to maximise the effective sum rate under transmit-power and power constraints.

6.2.3 Nonlinear optimisations

Robust design gets its roots from the control theories. It is recommended that a new look be taken at the robust beamformer design from the nonlinear or the robust control theories like H^{∞} [108], [109], and employing nonlinear optimisations, like Penalty/Barrier methods.

6.2.4 Multi-antenna users

An assumption used to develop beamforming schemes in this thesis is that user terminals are equipped with single antenna. When user terminals and base stations both have multiple antennas, there are more degree of freedom to effectively control interference. However, transmit and receive beamforming should be jointly designed. A question arising here is whether global optimality can be achieved by iteratively optimising transmit and receive beamforming. Complexity and signaling overhead are expected to significantly increase. Therefore, practical solutions to the optimal beamforming and tradeoff between optimality and complexity are open problems for research.

6.2.5 Non-flat fading channels

It is usually assumed that the system is designed to act in a flat fading environment. In rapidly changing environments it is not a practical assumption. It is recommended that the beamformer be designed for non-flat fading channels.

6.2.6 Cross-layer interference mitigation

For cognitive-primary interference mitigation, the proposed precoding schemes are based on null-shaping. It actually tightens the cognitive-primary interference constraint by reducing the interference limit. This is suboptimal in terms of the resulting CR throughput when the primary network is interference tolerant. Therefore, it is desirable to investigate other better precoding schemes without tightening the cognitive-primary interference constraint. As for cross-layer interference mitigation, it is worthwhile to take other interference management mechanisms like power or contention control of CR networks into the cross-layer optimisation.

6.2.7 Soft frequency reuse schemes

Soft frequency reuse is a special type of universal frequency reuse, in which cell edge users are transmitting with high powers compared to cell center users. All resources are allocated in the cell-center of each cell whereas a small part of the resources is allocated to the edge users of a cell. Based on the theoretical approach developed in this thesis, interference models for soft frequency reuse with different scheduling schemes can be derived with the critical performance analysis in terms of spectral efficiency and energy efficiency.

Appendix A: Proof of channel covariance matrix

Substituting for $\mathbf{h}_{i_s n}$ and $\mathbf{h}_{i_s t}$ in (3.8), one can calculate the (k,m) element of the channel covariance matrix of user i_s as

$$\mathbf{R}_{i_s}[k,m] = \mathbf{E}\left[\sum_{n=1}^{Q} c l_{i_s n}^{-\frac{\alpha}{2}} g_n e^{-j\beta_n} e^{-j\frac{2\pi}{\lambda}\left[d(k-1)\sin\left(\theta_{i_s}^{q}+\phi_n\right)\right]} \right.$$
$$\left. \times \sum_{t=1}^{Q} c l_{i_s t}^{-\frac{\alpha}{2}} g_t e^{j\beta_t} e^{j\frac{2\pi}{\lambda}\left[d(m-1)\sin\left(\theta_{i_s}^{q}+\phi_t\right)\right]}\right]. \tag{1}$$

Since the channels between the secondary BS and two different scatterers, i.e., $n \neq t$, are independent from one another, $\mathbf{E}\left(g_n e^{j\beta_n} g_t e^{-j\beta_t}\right) = 0$ for $n \neq t$. Otherwise, for the channel between the secondary BS and the same scatterer, i.e, $n = t$, one can write $\mathbf{E}\left[|g_n|^2\right] = \delta_g$, where $|g_n|^2$ has Chi-square distribution with two degrees of freedom. we can show

$$\mathbf{R}_{i_s}[k,m] = \sum_{n=1}^{Q} c l_{i_s n}^{-\frac{\alpha}{2}} \delta_g \mathbf{E}_{\phi_n}\left[e^{j\frac{2\pi}{\lambda}\left[d(m-k)\sin\left(\theta_{i_s}^{q}+\phi_n\right)\right]}\right]. \tag{2}$$

. APPENDIX A: PROOF OF CHANNEL COVARIANCE MATRIX

Assuming normal distribution for the angular spread, i.e.,$\phi_n \sim \mathcal{N}(0, \sigma^2)$, we calculate the expectation in (2) as follows

$$\begin{aligned}
\mathbf{E}_{\phi_n}\left[e^{j\frac{2\pi}{\lambda}\left[d(m-k)\sin\left(\theta_{i_s}^q+\phi_n\right)\right]}\right] &= \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{\frac{-\phi_n^2}{2\sigma^2}} e^{\frac{j2\pi d}{\lambda}\left[(m-k)\sin\left(\theta_{i_s}^q+\phi_n\right)\right]}\mathrm{d}\phi_n \\
&= \frac{e^{\frac{j2\pi d}{\lambda}\left[(m-k)\sin\theta_{i_s}^q\right]}}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{\frac{-\phi_n^2}{2\delta^2}} e^{\frac{j2\pi d}{\lambda}\phi_n\left[(m-k)\cos\theta_{i_s}^q\right]}\mathrm{d}\phi_n \\
&= e^{\frac{j2\pi d}{\lambda}\left[(m-k)\sin\theta_{i_s}^q\right]} e^{-2\left[\frac{\pi d\sigma}{\lambda}\left((m-k)\cos\theta_{i_s}^q\right)\right]^2}, \qquad (3)
\end{aligned}$$

where we have used the fact that $\sin(\theta+\phi_n) = \sin\theta\cos\phi_n + \cos\theta\sin\phi_n$ and ϕ_n is small such that $\cos\phi_n \approx 1$, $\sin\phi_n \approx \phi_n$. Furthermore, in arriving at (3), we have also used the following formula [110] with $p = \frac{1}{2\sigma^2}$, $n = 0$ and $q = \frac{j\pi d}{\lambda}\left[(m-k)\cos\theta_i^q\right]$

$$\int_{-\infty}^{\infty} x^n e^{-px^2+2qx}\mathrm{d}x = n! e^{\frac{q^2}{p}}\sqrt{\frac{\pi}{p}}\left(\frac{q}{p}\right)^n \times \sum_{k=0}^{\lfloor n/2\rfloor}\frac{1}{(n-2k)!\,(k)!}\left(\frac{p}{4q^2}\right)^k,$$

Substituting from (3) in (2), we calculate

$$\mathbf{R}_{i_s}[k,m] = \sum_{n=1}^{Q} c l_{i_s n}^{-\frac{\alpha}{2}}\delta_g e^{\frac{j2\pi d}{\lambda}\left[(m-k)\sin\theta_{i_s}^q\right]} \times e^{-2\left[\frac{\pi d\sigma}{\lambda}\left((m-k)\cos\theta_{i_s}^q\right)\right]^2}. \qquad (4)$$

Similarly, by changing the secondary index i_s to the primary index i_p in the above formulations, one can derive $\mathbf{R}_{i_p}[k,m]$ for the primary user. This concludes the proof.

Appendix B: Proof of Shur Compliment

The Schur complement of a matrix block [6], i.e., a submatrix within a larger matrix is defined as follows. Suppose $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are respectively $p \times p, p \times q, q \times p$ and $q \times q$ matrices, and $\mathbf{D}$ is invertible. Let

$$\mathbf{M} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix},$$

so that $\mathbf{M}$ is a $(p+q)(p+q)$ matrix.

Then the Schur complement of the block $\mathbf{D}$ of the matrix $\mathbf{M}$ is the $p \times p$ matrix

$$\mathrm{S} = \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}.$$

Applying the Schur compliment, we can write the first constraint ,i.e., $\mathbf{A}$ in (4.23) as

$$\sqrt{1+\frac{1}{\gamma_i}\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_i} - \frac{1}{\sqrt{1+\frac{1}{\gamma_i}(\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_i)}} \begin{bmatrix} \mathbf{e}_i^T\mathbf{H}_s\mathbf{W} & \sqrt{\beta_i} \end{bmatrix} \begin{bmatrix} (\mathbf{W}^H\mathbf{H}_s^H\mathbf{e}_i) \\ \sqrt{\beta_i} \end{bmatrix} \geq 0,$$

multiplying both sides of the above inequality by $\sqrt{1+\frac{1}{\gamma_i}\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_i}$, results to

$$1 + \frac{1}{\gamma_i}\left|\mathbf{e}_i^T\mathbf{H}_s\mathbf{W}\mathbf{e}_i\right|^2 \geq \mathbf{e}_i^T\mathbf{H}_s\mathbf{W}(\mathbf{H}_s\mathbf{W})^H\mathbf{e}_i + \beta_i,$$

furthermore

$$1 + \frac{1}{\gamma_i} \left| \mathbf{e}_i^T \mathbf{H}_s \mathbf{W} \mathbf{e}_i \right|^2 \geq \left\| (\mathbf{H}_s \mathbf{W})^H \mathbf{e}_i \right\|^2 + \beta_i,$$

is proved to be equal to the first constraint in (4.22). Similarly for the second constraint ,i.e., **B** in (4.23), we can write the Schur compliment as

$$\sqrt{I_m} - \mathbf{e}_m^T \mathbf{H}_p \mathbf{W} \frac{1}{\sqrt{I_m}} \mathbf{I} \mathbf{W}^H \mathbf{H}_p^H \mathbf{e}_m \geq 0,$$

multiplying both sides of the inequality by $\sqrt{I_m}$, we have

$$I_m \leq \mathbf{e}_m^T \mathbf{H}_p \mathbf{W} (\mathbf{H}_p \mathbf{W})^H \mathbf{e}_m,$$

furthermore

$$\sqrt{I_m} \leq \left\| \mathbf{W}^H \mathbf{H}_p^H \mathbf{e}_m \right\|,$$

which is equal to the second constraint in (4.22). The same solution is applied for the third constraint, i.e.,**C** in (4.23). This concludes the proof.

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